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TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA

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INVESTIGATION OF THE CONFORMAL GEAR FOR HELICOPTER POWER TRANSMISSION

Task 1D121401A14414
Contract DA 44-177-AMC-101(T)

June 1964

prepared by:

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This report represents a part of a continuing U. S. Army Transportation Research Command research program for the investigation of new concepts of high-speed reducers for use as main transmissions in helicopters. The main efforts of this program are directed toward deriving a reduction unit or units, with a reduction ratio significantly higher (40:1 and above) than those of currently used transmissions, which would be more compatible with the high rotational speeds of aircraft turbine engines. With this objective in mind, the conformal contact (Wildhaber-Novikov gear tooth form) investigation was undertaken.

This command concurs with the contractor's conclusions reported herein. The results obtained from this specific study indicate that further research investigations must be conducted before an evaluation of conformal contact gearing for aircraft application can be made.

This command concurs with the contractor's recommendations, and the continuing program is scheduled on this basis.

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Task 1D121401A14414
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INVESTIGATION OF THE CONFORMAL GEAR
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HELICOPTER POWER TRANSMISSION

R-345

Prepared by

THE BOEING COMPANY
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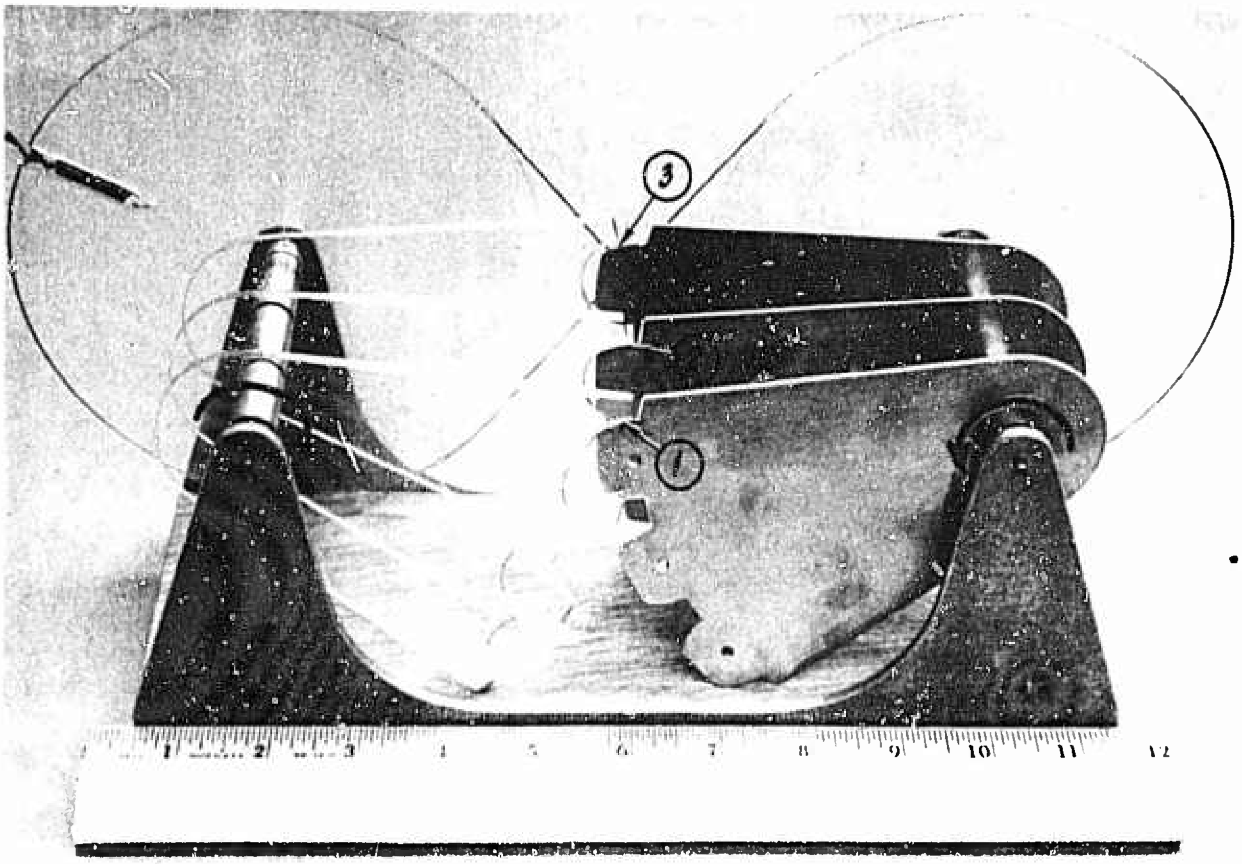

FIGURE NO. 1 CONFORMAL GEAR OPERATING PRINCIPLE

The conformal-contact (Wildhaber-Novikov) gear transmits constant angular velocity by successive contacts across the gear face. The laminates are to be visualized as parallel sections of one gear. Contact is shown at Point 1, with space at 2 and 3. If the gears rotate as shown, contact will occur next at 2 and later at 3. Each tooth contacts once during a full revolution. Sliding in the profile plane does not theoretically occur. The mating profiles can therefore be designed to conform and thus reduce contact stress as compared to the involute form.

SUMMARY

This report concludes Phase I of a study on new or improved methods of power transmission. This study is being performed by the Vertol Division of The Boeing Company for USATRECOM under Contract DA 44-177-AMC-101(T). The study was begun upon receipt of the contract on 28 June 1963.

Phase I comprises an analytical study of conformal contact gearing to determine the operational range for which the concept is most applicable to current or projected Army aircraft. The conformal contact or Wildhaber-Novikov gear tooth form was recommended for study by the contractor. Published literature on this type of gear has indicated considerable increase in load-carrying capacity as compared to involute gearing. Since the contractor is fully aware of the need for continued improvement in power transmission for VTOL and STOL aircraft, it was felt necessary to explore the potentialities of the conformed contact gear tooth. The desired advantages, as compared to existing final reduction systems, are as follows:

1. Improvement in power-to-weight ratio.
2. Increased reliability.
3. Ability to accept higher input speeds.
4. Improved efficiency.
5. Increased simplicity.
6. Ultimately a lower unit production cost.

Phase I has been subdivided into: (1) an analytical investigation of the conformal tooth bending and contact

stress, and the effect of various geometrical changes; and (2) a design investigation, using the analytical method, to illustrate the advantages and problems of the conformal tooth transmission for various ratios and horsepower inputs.

As a separate and independent effort, the Vertol Division has conducted a photoelastic investigation to reinforce the analytical work conducted under contract.

CONCLUSIONS

1. The comparison between the conformal contact transmission and the involute planetary shows theoretical advantages for the conformal contact transmission.

2. These advantages may be summarized as follows:

   a. Total effective weight is less, if the power loss per mesh is considered equal in both types.

   b. The inherent reliability is greater by reason of the open-mesh and the dual-mesh characteristics of the conformal transmission.

3. The practical problems of the conformal transmission, as analyzed and pictured, lie chiefly in the face-to-diameter ratio of the pinions. The analysis indicates that the load-carrying advantages of the conformal gear are realized only as the pinion face-width-to-diameter ratio approaches one. This necessity, if proven by test, would make for serious problems in adequately supporting the conformal gear and in providing helix angle correction to ensure uniform tooth loading from driven-end to free-end.

4. The real advantages of the conformal contact gear, for V/STOL transmission systems, must be assessed by tests of actual gears, run under load. The tests, if successful, would demonstrate that the extreme face width indicated by analysis is not in fact required.

5. It is considered that further work aimed at detail design of an aircraft type transmission would merely underscore the problem areas, which have been defined, without providing solutions to them.

RECOMMENDATIONS

1. The conformal contact gear should be load-tested to provide basic data for further design evaluations.

2. The test specimens should as nearly as possible represent the best current aircraft manufacturing practice as applied to involute gears. The remainder of the setup should utilize existing test equipment, not necessarily simulating aircraft practice in material or stress level.

3. Using this approach, gear failure data can be taken expeditiously, economically, and without the probability of housing, shaft and bearing damage occurring at every failure. This failure data should be compared to involute gear failure data, taken as nearly as possible in the same manner and environment. The comparison will provide an index number, or rating factor, which will then be used in detail design of an aircraft unit.

4. It is apparent that the successful application of gearing with increased load-carrying capacity will subject bearings to proportionally higher loads. To realize the benefits of such gearing, the associated application of higher capacity bearings will be required. It is therefore recommended that the hydrodynamic bearing be the subject of parallel research and development, in particular toward providing this type of bearing with a capability of emergency no-oil operation without catastrophic failure. It is also considered necessary to investigate the power-loss characteristics of this type of bearing, and to determine how efficiency can be improved.

INTRODUCTION

The need for a power transmission system capable of matching the speed, size and weight characteristics of the gas turbine resulted in the funding by the Army of Contract DA 44-177-AMC-101(T).

Initial investigations in the area of power transmission systems indicate that the engine rpm should be reduced at the rotor terminus to obtain the best specific weight for the system.

To achieve this goal, a high-ratio, high-efficiency reduction mechanism is required. It was with this objective in mind that a study of the conformal contact (Wildhaber-Novikov) gear tooth form was proposed in Boeing-Vertol PR-445*.

The first description of this form of gear tooth was by Ernest Wildhaber, in a United States patent filed in 1923 and issued October 5, 1926. This patent contains essentially every feature of the conformal contact gear being investigated today. Wildhaber describes circular arc profiles in both normal and transverse planes, and also the differences between concave and convex profiles to allow change in center distances. This last modification is generally credited to M. L. Novikov, who received a U.S.S.R. Patent in 1956. The chief value of Novikov's work was to revive interest in the gear form. As it has frequently happened, the necessity, in this instance a requirement for high-duty V/STOL transmissions, came long after the invention.

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* The Boeing Company, Vertol Division, Proposal For Development Of Helicopter Drive Systems, PR-445, 1 May 1963.

The Wildhaber-Novikov gear form uses convex and concave surfaces on mating teeth to create a band of contact, which spreads to area contact under load. By comparison to involute teeth, the conformal shape is not conjugate in the plane of rotation. For constant velocity transmission, the gear is made helical. The contact runs axially along the face as the mating gears revolve. To operate without interruption, the gear teeth must have overlap; that is, the face width must be sufficiently wide to include at least one, and preferably more, axial pitch. The geometric variables of the conformal gear tooth can be manipulated to a considerably greater extent than the involute tooth. Aside from pitch and pressure angle, which can be chosen as in the involute form, the profile radius, tooth height, root shape, overlap, and tooth thickness are susceptible of variation to balance bending and contact stresses between gear and pinion.

The common form of conformal contact uses a circular arc tooth profile, where the pinion is convex and the gear concave. To permit some variation in operating center distance without edge loading, the concave tooth radius is somewhat larger (2 to 10%) than the convex. The pinion is generally all addendum with the center of profile radius described from the pitch diameter. The gear is all dedendum with its profile radii described from the pitch diameter.

The study work performed under contract has dealt with three areas: technical review, analysis (in which a method for stress estimation of the conformal gear was developed), and design (in which the results of the analysis were used in various arrangements). The problems and advantages of the arrangements were investigated. Results of this investigation were plotted. Further investigation was performed as a Vertol independent research effort. This was the photoelastic study used to supplement the analytical work performed under contract. So far as is known, this photoelastic study is the first attempt made to visualize conformal contact gear stresses.

METHOD

TECHNICAL REVIEW

A literature search was conducted on conformal contact gearing (see Bibliography). As an additional preliminary step, personal contacts were made or renewed with those in the field. The following individuals and companies were interviewed for their experience with conformal gearing:

Mr. C. F. Wells	(Manager, Gear Engineering Department, Heavy Plant Division, Associated Electrical Industries, Rugby, England)
Mr. A. M. Gunner	(Gear Engineer, also with AEI)
Mr. Wells Coleman	(Chief, Gear Analysis, Gleason Works, Rochester, N. Y.)
Mr. W. J. Davies	(Aero Division, Rolls Royce, Rugby, England)
Mr. E. J. Wellauer	(Director, Research and Development, Falk Corporation, Milwaukee)
Dr. A. Seireg	(Professor, Marquette University, College of Engineering, Milwaukee, Wisc.) Retained as consultant

Associated Electrical Industries (AEI) produces the Circarc gear, which follows the Wildhaber-Novikov system of conformal contact.

Most of AEI's work has been with unhardened, moderate-strength steel gears (Brinell numbers of 233 to 277), at fairly low speeds. Normal applications are in the 3000-rpm input range. An investigation of higher speed uses (up to 20,000 rpm) is now in process. AEI believes that the Circarc system will raise load factors 3 to 4 times over their normal involute gears. They are investigating non-roller bearing applications, where the center distance change may be greater. Mr. Wells stated that there was some improvement in efficiency, as compared to their involute gears, and that the oil film was perhaps five times thicker. He mentioned that Westland Aircraft was working on this system, using titanium gears which AEI expected to cut for them. No major change in noise or vibration was expected.

Mr. Gunner made a visit to Vertol on 27 September 1963. He discussed his views of AEI Circarc gearing. His advanced project at that time was design and fabrication of a 3200-hp gear set for a compressor drive. This design was predicted to carry approximately twice the load per inch of face that their involute experience would indicate permissible. According to Mr. Gunner, no side-by-side test to failure of Circarc and comparable involute gearing had been performed. He was not aware of any rigorous analytical technique at AEI to design Circarc gearing; rather, they had relied on testing to obtain their design information.

Mr. W. J. Davies of Rolls-Royce stated in July 1963 that they were about to start work on a 5,000-hp, 15,000-rpm-input gear set using the Novikov profile. They are planning to duplicate the "Tyne" reduction gearing, which normally has a 6.50-inch center distance, with a much smaller Novikov unit of 3.25-inch center distance. A 10-degree helix angle and 20/61 teeth will be used. Face width will be considerably wider than now used. Materials and fabrication will be exactly as in the normal "Tyne" gears. Davies expressed the opinion that the teeth are so good that it is difficult to provide

support and minimize deflections to give the reductions in size and weight at first expected. Contrary to Wells, he thought that noise and vibration might be worse than with comparable involute systems.

ANALYTICAL METHOD

The analytical effort, performed by Vertol under contract, undertook to determine the bending stress and the surface contact stress of the conformal contact tooth. The proper balance of these stresses produces a gear with compatible beam strength and wear resistance. The aircraft involute gear has from experience been optimized in geometry within the limitations imposed by the involute form and the necessity for conjugate action in the profile plane. It is not unusual to find such a gear designed to allowable limits for both types of stresses. The conformal contact gear, in contrast, offers more freedom in the choice of geometric variables since it is not a conjugate gear. The superiority of the conformal gear exists insofar as it is possible to balance stresses at a higher load level than the involute gear. The increased contact capacity available through profile conformity gives reason to believe that such an improved load level can be achieved.

Bending Stress

The tooth bending stress depends upon the tooth load and upon the distribution of that load. In the profile plane, when the radii match, the conformal contact system produces a line load perpendicular to the axis. A point load is produced when the radii differ. In the axial plane, the curves of the helixes oppose and produce a point loading. When elastic deformation is neglected, therefore, the area of loading is normally a point.

Using the Hertz equations for bodies in contact, the area of loading can be determined for a practical case where deformation exists. The two radii of the contact

body are calculated; the "equivalent" radius of a curved body in contact with a plane is then determined. This equivalent radius is dependent upon the helix angle, the ratio, the size of gear, and the height of the tooth. Figure 2 illustrates this. From the equivalent radius, the band width of contact axially along the tooth face is obtained.

To relate this to stress at the tooth root, the Wellauer-Seireg paper was applied. This paper deals with the stress distribution at the base of a cantilever plate. A plate is differentiated in this case from a beam, in that the load is not uniform along its (spanwise) length, and that its aspect ratio is 4 to 1 or more. A concentrated load produces a moment distribution curve of a specific shape along the fixed edge. This shape has been determined analytically and verified experimentally. The conformal gear problem is to determine the moment intensity under load patterns which extend for various spanwise distances, and in which the load distribution is assumed elliptical. The load span is dependent upon the mating radii of the gear, and also upon the load deformation. A given gear set will enlarge its loading band as the tangential load is increased. The effect of this is a nonproportional increase in root bending, nonproportional because as the load is increased, it also diffuses further into the hitherto unstressed root area. A correction factor for bending stress (Ki) was obtained and plotted (Figure 3). It can be seen that as contact band-to-total-face-width ratio increases, the correction factor and the bending stress decrease. From a concentrated load to a full-width load, the expected change in bending stress is 3 to 1. However, the majority of study examples had load-width-to-total-width ratios of about one quarter.

When the bending stress equation is examined, one additional factor is found in addition to the Ki distribution factor. This factor is Kf, the stress concentration factor. A conventional equation, which accounts for

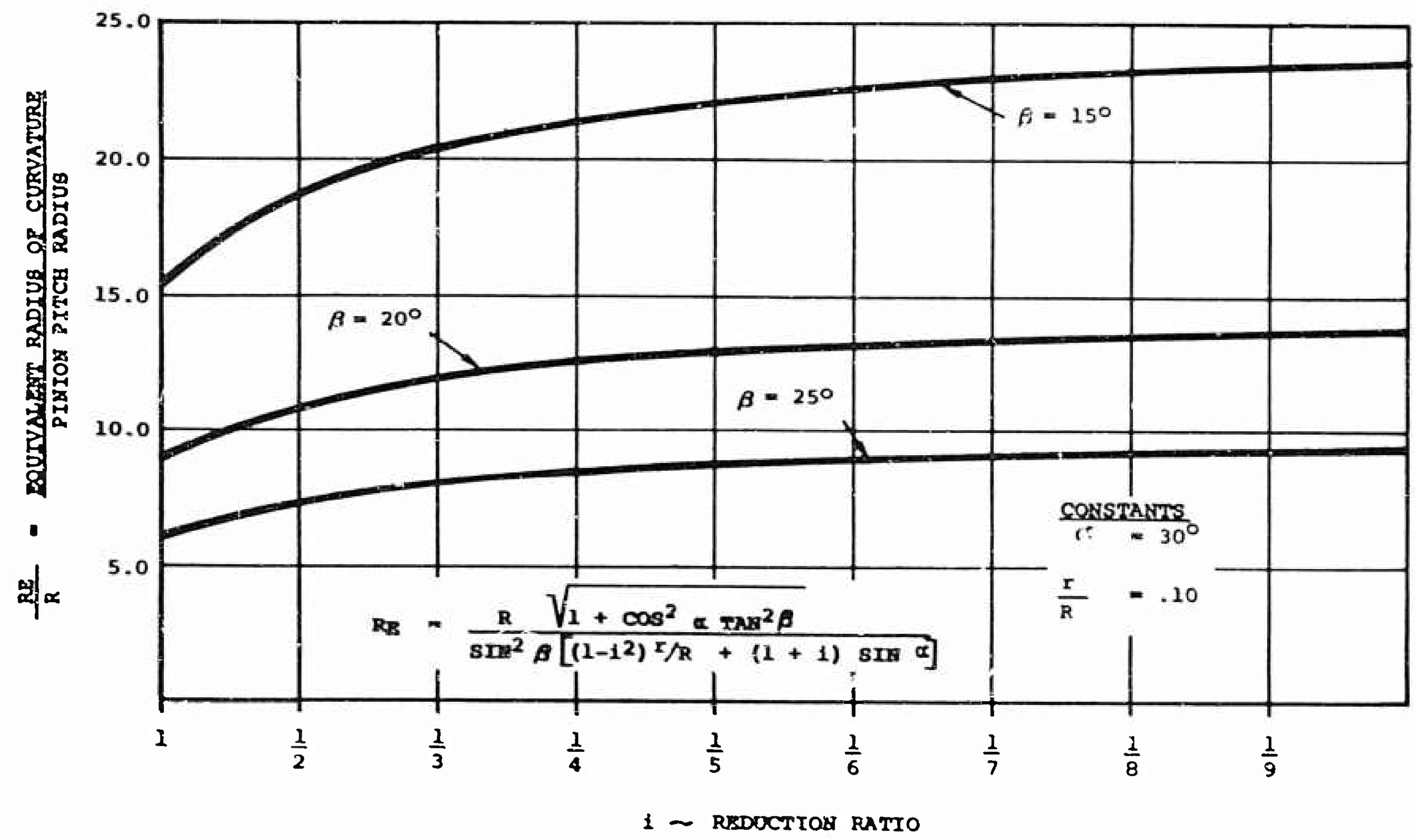
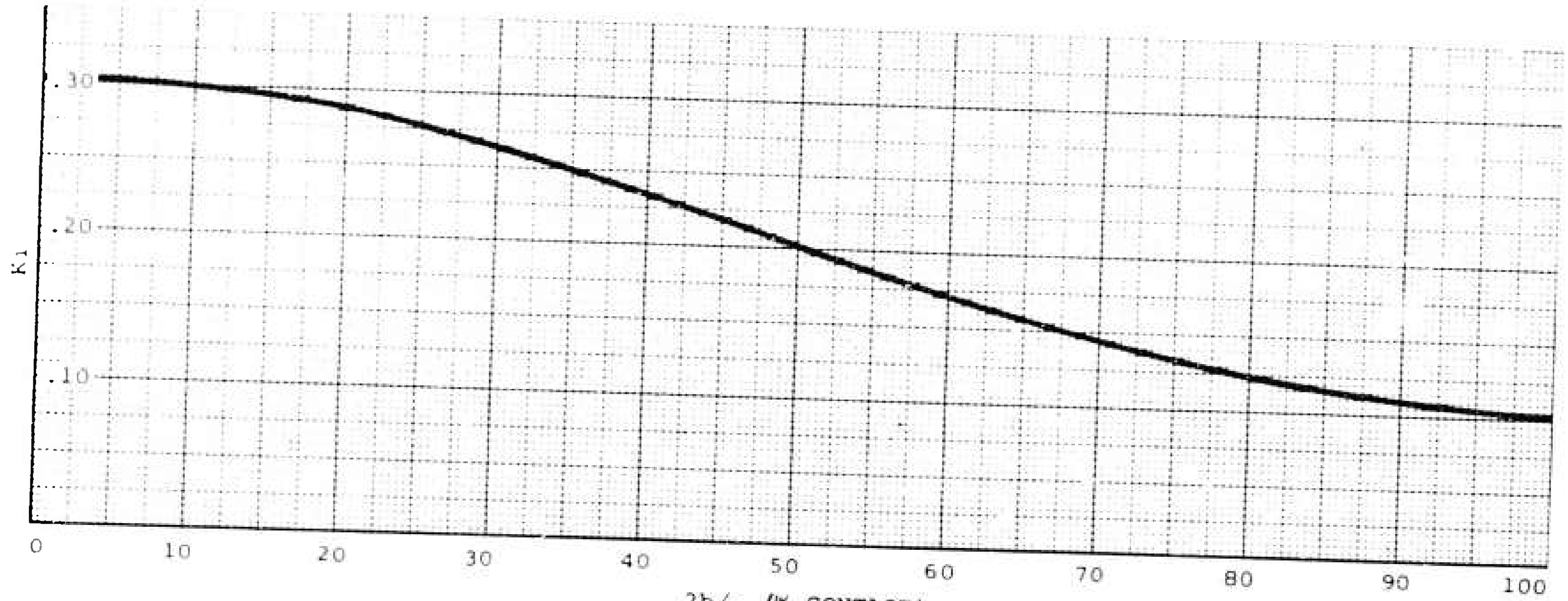


FIGURE NO. 2 EQUIVALENT RADIUS VS GEAR RATIO

$\frac{\text{MAXIMUM MOMENT}}{\text{AVERAGE MOMENT}}$ (Ki) VS % CONTACT $(\frac{2b}{F})$



2b/F (% CONTACT)

F = FACE WIDTH 2b = WIDTH OF CONTACT BAND

FIGURE NO. 3 MOMENT FACTOR VS CONTACT RATIO

various beam proportions and root radii, was used initially. To further determine this factor, the photoelastic technique was used with varying root shapes. The photoelastic results agree generally with the analytical factor.

To complete the bending stress determination, the radial compressive stress was calculated. This is caused by the tangential tooth load acting upon the inclined tooth face. The effective area was assumed as the band width having the same elliptical distribution as previously. Acting as a bending moment relief, this radial component adds to the stress on the compression side of the beam and subtracts from the tension side. Its effect is therefore beneficial since tooth fatigue failure occurs on the tension side, due to the characteristic lower endurance limit.

Contact Stress

The contact stress assumes the same elliptical load distribution over the band of contact. The other axis of the contact area is assumed as the normal height of the tooth profile. Full conformity is thus used; this condition will not actually pertain particularly at partial load. However, less than full conformity will increase the elastic deformation in the spanwise (axial) direction, and so increase the length of band of contact. It is therefore considered that this assumption is sufficiently realistic to be a useful approximation.

The influence of the tooth geometric variables upon the bending and contact stress levels was considered. It was apparent that the relationship between bending stress and contact stress was dependent upon tooth thickness for beam strength and dependent upon tooth profile radius and helix angle for area of contact. The balance of tooth bending and contact stress is an important step in optimizing the gear. The results of detail studies, in which one variable was changed and the stresses determined, pointed out that beam strength was the limiting

factor. The contact capacity of the conformal gear is generally superior to the bending capacity, unless the gear tooth thickness is increased to extremes by normal involute standards.

The following equations were used in this study to obtain bending and contact stresses:

TO FIND NORMAL LOAD:

$$P_N = \frac{P\sqrt{1 + \cos^2\alpha \quad \tan^2\beta}}{\cos\alpha}$$

Where: P_N = load normal to face

P = tangential load = $\frac{\text{Torque}}{\text{Pitch Radius}}$

α = pressure angle

β = helix angle

TO FIND HEIGHT OF CONTACT BAND:

$$L = 2 \sin \alpha \; r$$

$$L_N = L\frac{\sin \alpha}{\sin \alpha_N}$$

$$\tan\alpha N = \tan\alpha \cos\beta$$

Where: L = height of face in transverse plane

L_N = height of face in normal plane

αN = normal pressure angle

r = radius of tooth profile

TO FIND LENGTH OF AXIAL CONTACT BAND:

$$2b = 2.15 \sqrt{\frac{2 P_N R_E}{E L_N}}$$

Where: $2b$ = length of band

R_E = equivalent radius (Figure 2)

E = Young's modulus

TO FIND BENDING STRESS:

$$fb = \frac{6 Ki Kc P_N \cos\alpha \quad N}{(T'N)^2} - \frac{P_N \sin}{2T'N b}$$

Where: fb = bending stress at tension fillet

Ki = correction factor (Figure 3)

Kf = concentration factor

$T'N$ = critical section in normal plane

b = half length of axial contact band

TO FIND CONTACT STRESS:

$$fc = \frac{4}{\pi} \frac{P_N}{2b L_N}$$

Stress Balance

To change tooth thickness, the gear designer may manipulate three variables: overlap ratio, helix angle, and face width. Bending strength is thus balanced to contact capacity.

$$\text{Diametral Pitch} = \frac{\text{Overlap} \times \pi}{\text{Face} \times \tan \text{Helix Angle}}$$

$$\text{Thickness} = \frac{K}{\text{Diametral Pitch}}$$

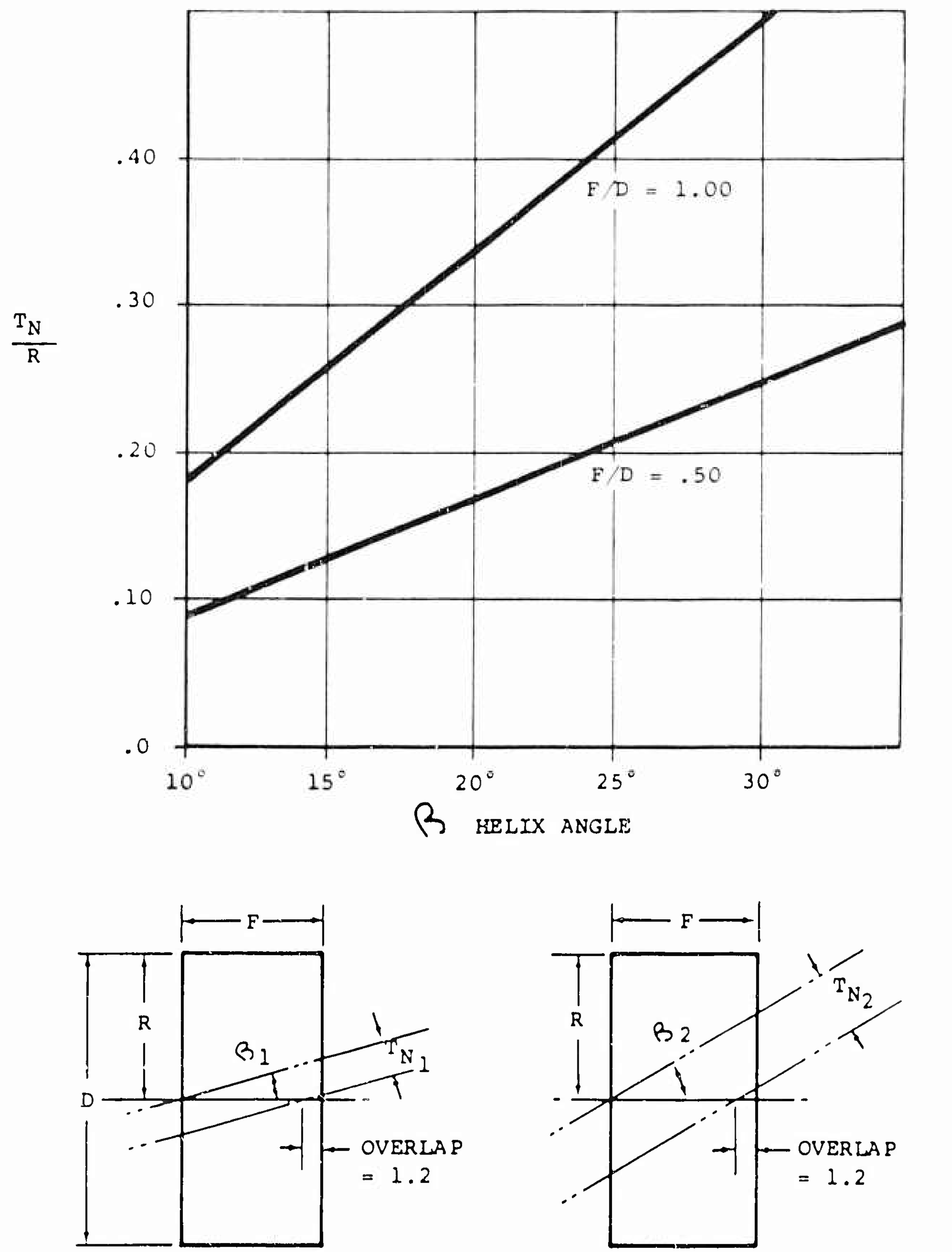
Where: $K = 1.85$ when $T_1/T_2 = 1.5$

Figure 4 illustrates the effect of helix angle on tooth thickness.

A minimum overlap ratio of 1.0 is required, and 1.2 is recommended in the literature as the minimum to reduce tooth end load effect. Wellauer and Seireg give empirical support to this figure in their paper. A 1.2 ratio was held throughout the study. A greater ratio might be used if failures are found to occur at the end of the tooth. The tooth thickness is reduced however, when overlap is increased. Selection of a minimum overlap ratio must therefore be verified by examination of tooth failure origins.

Face width is the next parameter. A maximum face-to-diameter (F/D) ratio of 1.0 was selected for this study. Recent experience includes spur gear F/D ratios as high as 0.8. Generally, F/D ratios higher than 1.0 will not provide capacity in proportion to the face width because of torsional and bending deflections. As noted in the design discussion, the F/D of 1.0 gives problems of adequately supporting the pinion.

The final parameter, helix angle, remains to supply the increased tooth thickness. As the helix angle increases, the band of contact decreases and contact stresses [illegible]se. Tooth thickness increases and bending stresses decrease by the square of the thickness. There is, therefore, a tendency to increase helix angle in the study configurations until bending strength is compatible with contact capacity. It is interesting to note that much of the literature recommends a low helix angle. This is under-



INCREASED HELIX ANGLE RESULTS IN INCREASED TOOTH THICKNESS T_N

FIGURE NO. 4 TOOTH THICKNESS VS HELIX ANGLE

standable in view of the relatively soft gear material referred to. In those cases, common to non-case-hardened industrial and marine gearing, wear (surface stress) is the design limitation. A low helix angle can most certainly increase this wear capacity, although whether to the extent of 400 to 500%, as claimed, is questionable.

DETERMINATION OF DESIGN ALLOWABLES

It is recognized that calculated gear stresses are index numbers, rather than absolute values which can be directly correlated to simple beam fatigue testing. Allowable stresses are empirically determined for a general application; they are not directly transferable from one type of gear to another.

A comparison of involute planetary systems and the proposed conformal-contact non-planet reduction was used to provide a preliminary estimate of allowable tooth bending and contact stresses.

Considerations which would imply higher permissible stresses are:

1. The bending stress does not undergo reversal in the conformal transmissions studied. In the planetary gear the mesh occurs alternately on opposite sides of the tooth. The alternating stress is therefore twice the single mesh stress. The fatigue endurance limit is reduced by this more severe condition, according to the Goodman diagram. This contrast should provide a higher bending allowable for the non-planet gear. The estimated improvement factor is approximately 1.4.

2. It is considered by many references that an oil film of increased thickness is likely in the conformal mesh as compared to the involute.

The effect would be to dampen the irregularities of tooth spacing and profile and to enlarge the load carrying area. This would reduce dynamic bending stress and reduce contact stress.

3. The contact stress allowable may be higher than that for involute gears. There is an apparent reduction in sliding velocity. The literature of combined rolling and sliding tests indicates that a mechanism in pure rolling contact can withstand higher pressures, without distress, than is possible when sliding is added. This infers a superiority for the conformal gear.

Considerations which imply lower allowables are:

1. The face overlap ratio of an involute helical gear is often 2.0 or more than 2.0. Combined with profile overlap, this continuity of action assists in reducing dynamic loading as compared to the spur. Because of the conformal gear limited face overlap (1.2) and zero profile overlap, the assumption of lowered dynamic loading is not warranted. Also, the greater stiffness of the low, thick, conformal tooth may increase dynamic load from tooth inaccuracies, as compared to the involute gear.

2. The high face-to-diameter ratio theoretically predicted as necessary to develop advantages over the involute gear may introduce problems in equalizing load, which suggests a lowered bending allowable.

Table 1 summarizes the considerations leading to selection of the conformal design allowable:

TABLE 1

CONSIDERATIONS OF CONFORMAL DESIGN ALLOWABLES

Factor	Unidirectional Load	Helical Action	Tooth Thickness	F/D Ratio	Reduced Sliding	Increased Oil Film
Bending Stress	+	-	-	-		+
Contact Stress					+	+

\+ Indicates higher allowable
\- Indicates lower allowable

There is reason to believe that permissible stress levels, in both bending and contact, may be higher in the conformal gear. A quantitative examination of the increase seemed premature at this stage of analysis. For the designs and trends shown, therefore, gear stress levels comparable to those current were used. It is considered that these levels are conservative, as befits a relatively new concept, and that they are susceptible to improvement by development and testing.

DESIGN CHART FORMULATION

The conformal analysis was reduced to design charts (Figures 5 and 6) to aid in the preliminary design.

Figure 5 shows the allowable torque for various pinion sizes and for various face-to-diameter (F/D) ratios. It compares the involute gear and the conformal gear. From Figure 5, the involute gear is superior in load capacity

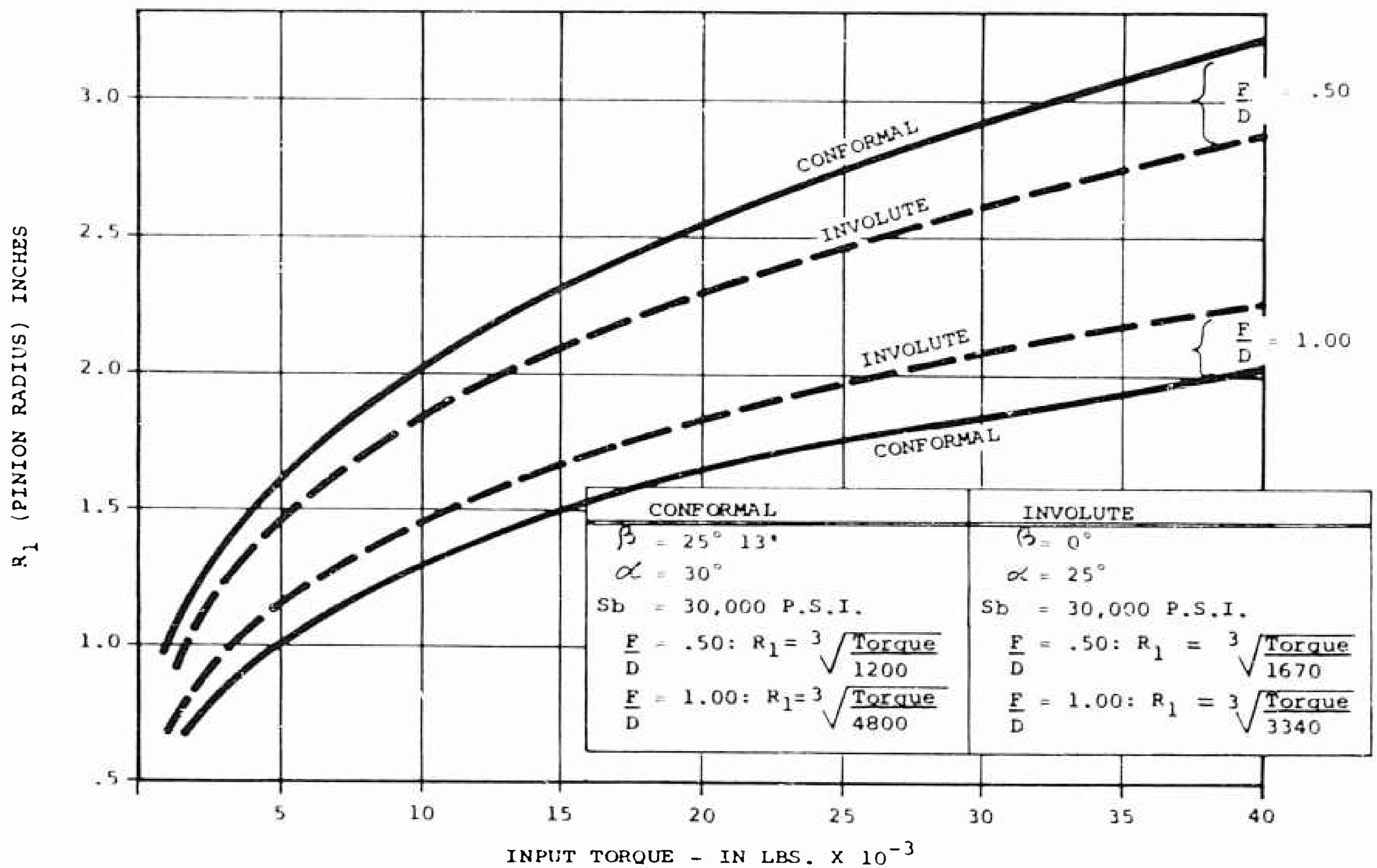


FIGURE NO. 5 PINION RADIUS VS TORQUE

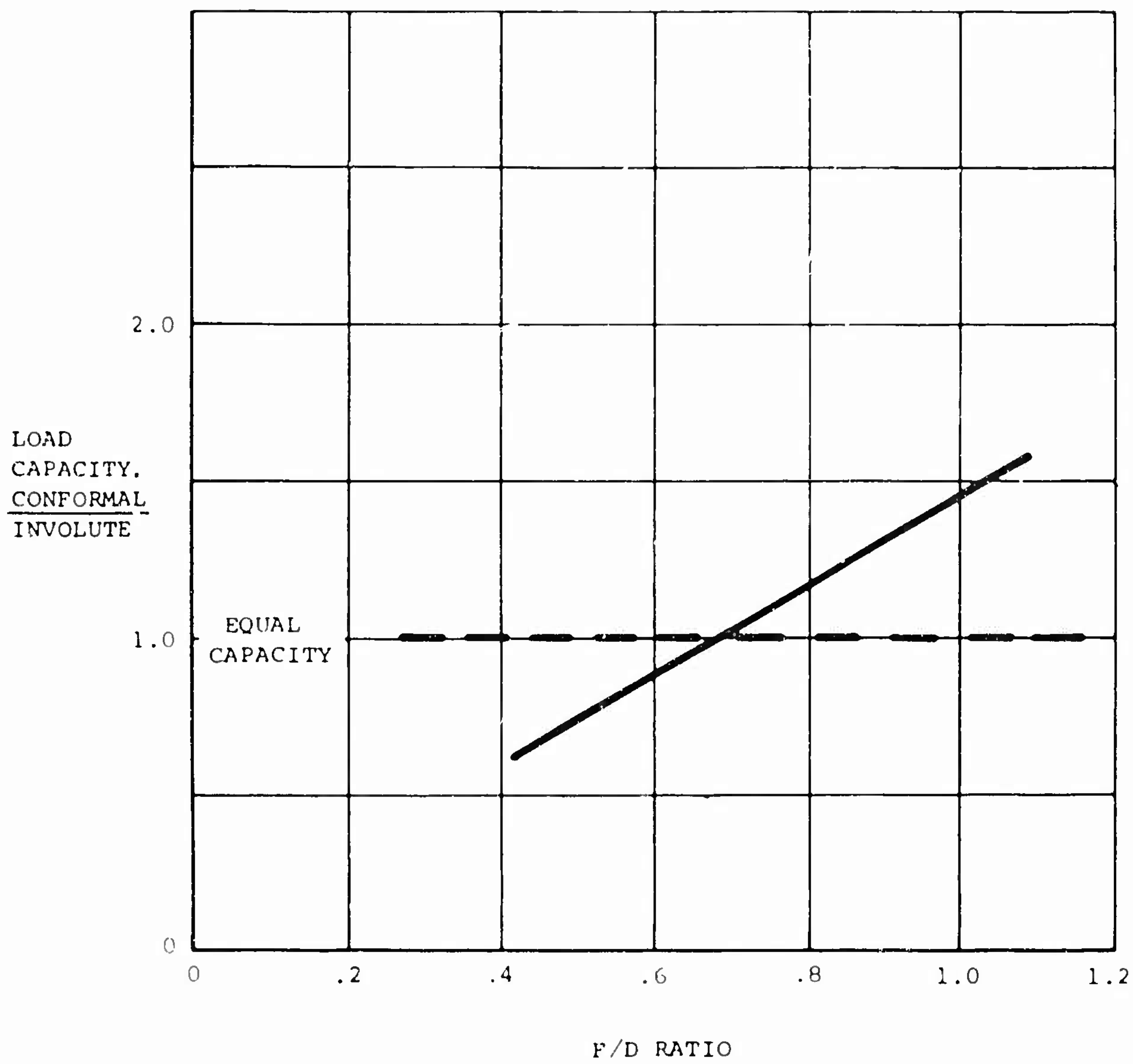


FIGURE NO. 6 LOAD CAPACITY VS FACE/DIAMETER RATIO

when F/D = .5; but conformal gear is superior at F/D = 1.0. Figure 6 shows the effect of various F/C ratios on the calculated capacity of both types of gears. The cross-over point is noted at an F/D ratio of .70.

The design curve was based upon the following:

1. It was assumed from previous work that the pressure angle would be 30° for maximum bending strength.

2. A maximum helix angle is desirable. Practical maximum appears to be 25° when the effect of thrust on bearings and gear webs is considered. For design simplicity, it is desirable to keep envelope sizes of radial bearing and thrust bearing approximately equal. This is generally possible at helix angles of 25° or less, assuming usual bearing spacings and 5:3 capacity ratio between roller and ball bearings.

3. To give an even number of teeth, the helix angle was modified to 25°-13' (Figure 7). This gives 8 teeth for F/D = 1
   12 teeth for F/D = .75
   16 teeth for F/D = .50
   using a 1.2 overlap ratio.

4. The normal tooth load P_N is obtained:

$$P_N = \frac{1.2 \text{ x torque}}{R_1} \qquad \text{Where } R_1 = \text{Pinion radius}$$

5. Rearranging the bending stress equation:

$$T_N = \sqrt{\frac{Ki\ Kf\ P_N \cos}{Sb \text{ allowable}}}$$

Where: T_N = normal tooth thickness at critical section

K_i = distribution factor, assumed = .25

K_f = concentration factor = 1.5

S_b allowable = design bending stress (30,000 psi)

= pressure angle = 20°

Then: $$T_N = \sqrt{\frac{P_N}{15{,}400}} = \sqrt{\frac{1.2 \times \text{Torque}}{15{,}400\ R_1}}$$

6. From photoelastic experience, T_N may be optimized to equal 80% of the circular pitch.

By substitution, $$T_N = \frac{2.47}{Pd}.$$

7. $$Pd = \frac{1.2 \times \pi}{F \times \tan\beta}$$

Where: Pd = Diametral pitch

1.2 = Overlap ratio

F = Face

β = Helix angle = 25°-13'

For the conditions specified,

$$Pd = \frac{8.04}{F}.$$

For a F/D ratio of .5, $F = R_1$ and

$$Pd = \frac{8.04}{R_1}.$$

Substituting from paragraph 6,

$$T_N = \frac{2.47\ R_1}{8.04} = .306\ R_1.$$

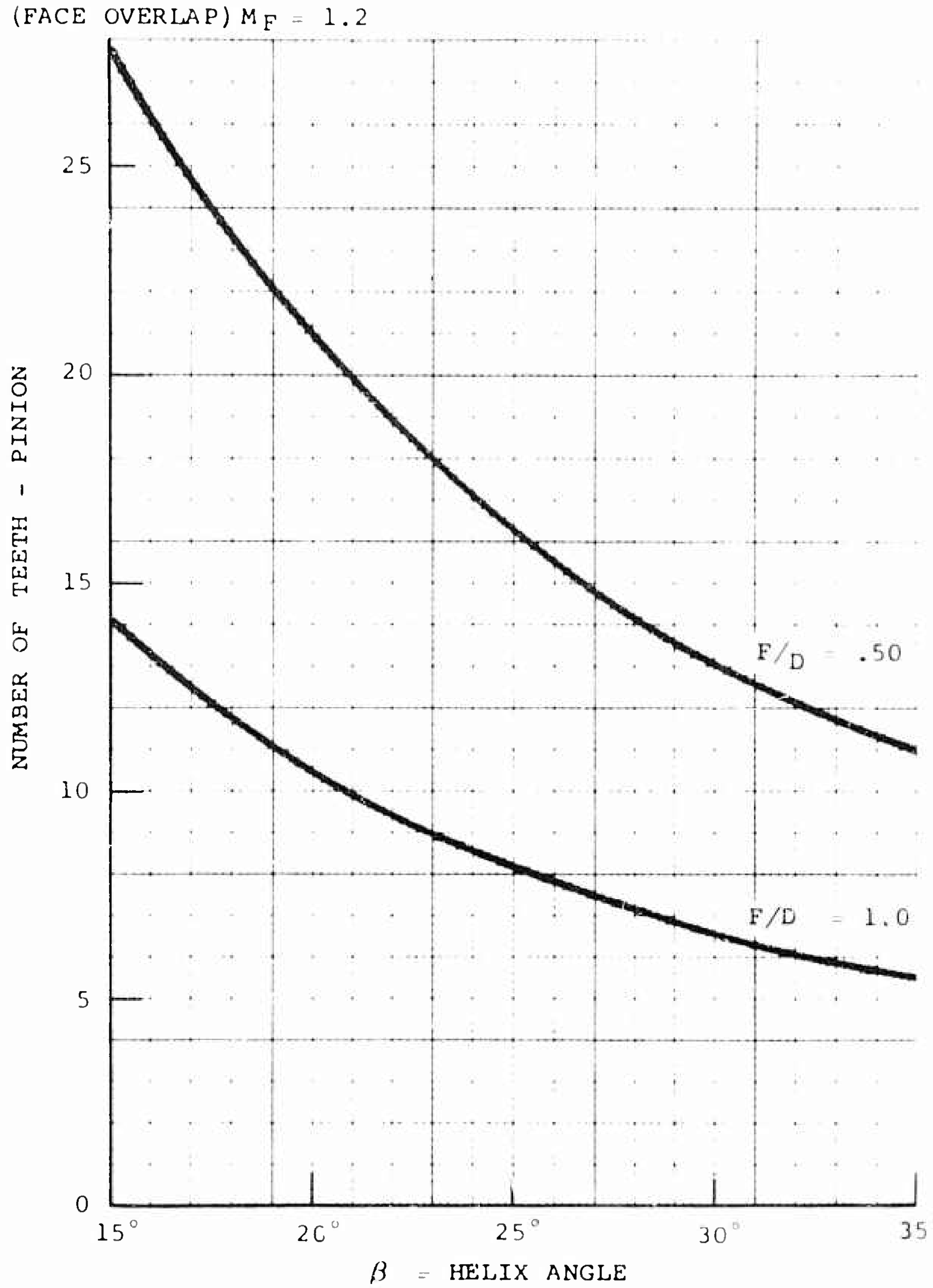


FIGURE NO. 7 NUMBER OF TEETH VS HELIX ANGLE

8. Returning to equation of paragraph 5,

$$R_1 = \sqrt[3]{\frac{Torque}{1200}} \; ;$$

generally, $$R_1 = \sqrt[3]{\frac{Torque}{C}} \; .$$

"C" may be regarded as a design constant; it is directly dependent upon F/D ratio and, hence, upon the helix angle. To show the effects of the geometric variables upon "C", Figure 8 is referenced.

This chart and Figure 5 are preliminary design aproximations used as an aid in estimating the gear size. The assumptions are considered valid within the investigated range of F/D ratio (.5 to 1.0) and helix angle (15° to 30°).

The design curve for the involute spur gear which is used as a comparison was derived as follows:

1. A practical minimum number of teeth was assumed for maximum bending strength.

$$N = 18$$

Diametral Pitch $Pd = \frac{N}{2R_1} = \frac{9}{R_1}$

2. The conventional AGMA bending stress equation was used:

$$sb = \frac{Wt \times Pd}{R \times Yk}$$

Where: Wt = Tangential load = $\frac{Torque}{R_1}$

$F = R_1$ for $F/D = .5$

Yk = form factor, generally = .50

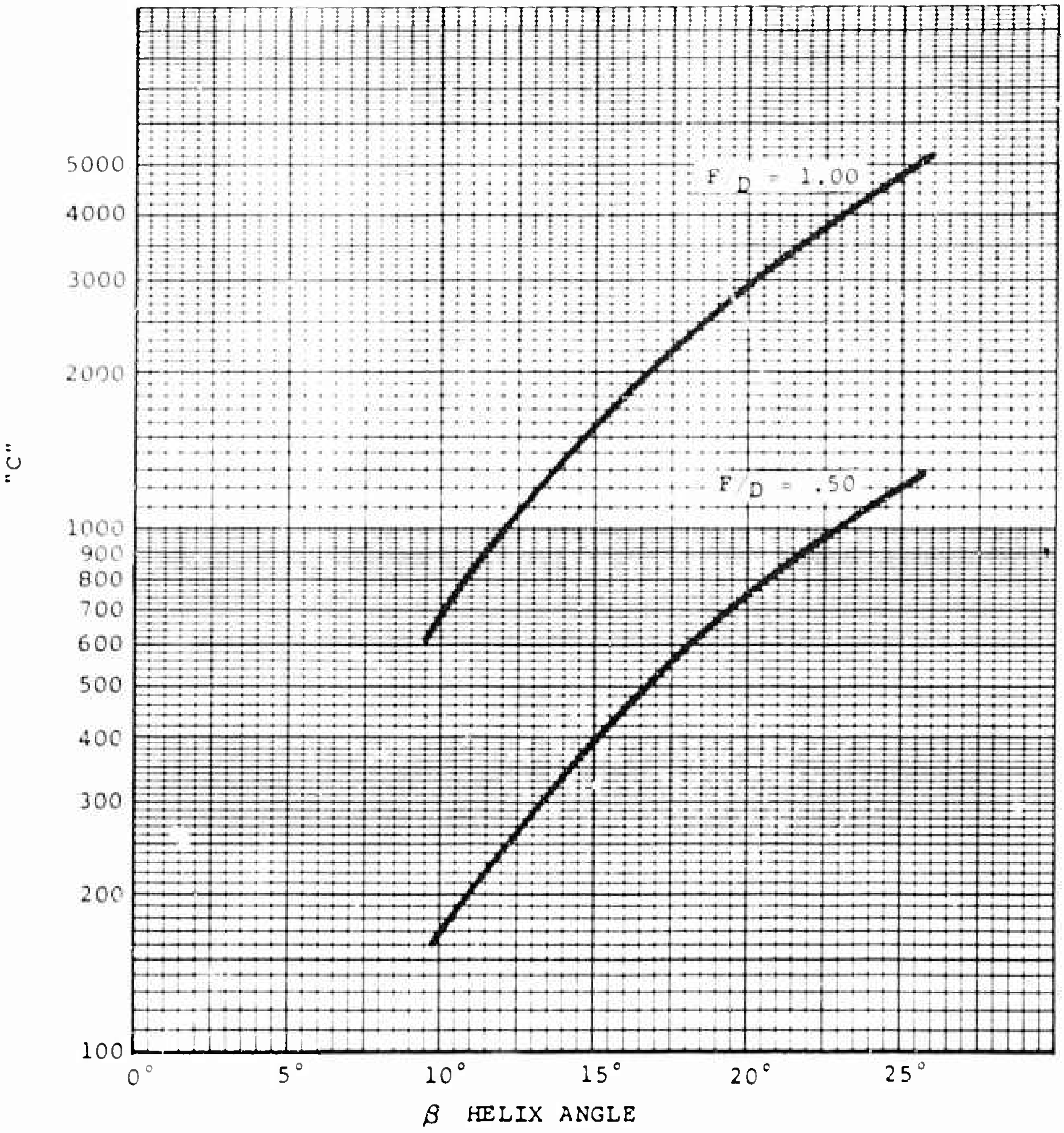


FIGURE NO. 8 DESIGN CONSTANT VS HELIX ANGLE

Then: $Sb = \frac{18 \times \text{torque}}{R_1^{\,3}}$

If Sb allowable = 30,000 psi, as in the conformal gear design,

$$R_1 = \sqrt[3]{\frac{\text{Torque}}{1670}}$$

3. The constant "C" (in this case 1670) is directly proportional to the face width (reference Figure 5).

PHOTOELASTIC ANALYSIS

To aid the analysis of the conformal gear, a Vertol independent research program was conducted in parallel to provide experimental data.

The objectives of this program were:

1. To compare geometric variables and to determine effect on tooth bending strength.

2. To establish basic design parameters, such as critical section, and to establish the concentration factor.

The approach to these objectives was by two-dimensional photoelastic models mounted and loaded in a special rectangular frame. Figure 9 illustrates the loading frame with models in position. The models were segments of gears, each with two teeth. They were supported by metal sandwich plates which contained the center hole. The models were enlarged to approximately 8:1 scale to compensate for the expected dimensional variations unavoidable with the material and the method of profiling from metal templates.

The tooth loading was observed under two conditions: first, with the profiles engaged normally giving a form

of area contact; second, with a .12-inch wide shim located at the line of action giving a concentrated load shown in Figure 10. The second approach was considered the more reliable because of better repeatability and more general application. Area contact was typical only of the particular photoelastic model with its individual profile inaccuracies and surface finish. The point contact eliminated these and compared the effect of the tooth shapes. However, the area contact method revealed a change in the position of load centroid as the tooth load was varied. The relative rigidities of the concave and convex teeth are believed responsible for this effect, which relates with published information on distress of the concave profile under high load. This load shift must be compensated for by an appropriate modification to the circular arc profile.

Six gear-tooth configurations were analyzed photoelastically. Each configuration comprised a concave segment and a convex segment. The configurations are tabulated in Table 2. All pressure angles were 30°. See Figure 11 for an explanation of symbols. The variables were chosen for the following reasons:

Pitch - To investigate the effect of thickness change on bending capacity.

T_1/r - To vary tooth height with respect to width (change aspect ratio).

T_1/T_2 - To investigate the balance of bending stress between concave and convex teeth. The literature recommends $T_1/T_2 = 1.5$.

Root Shape - To investigate the trade-off between a full round root and a flat root. The round root concentration factor is lower, but the added depth increases the bending arm and thus the moment at the critical section.

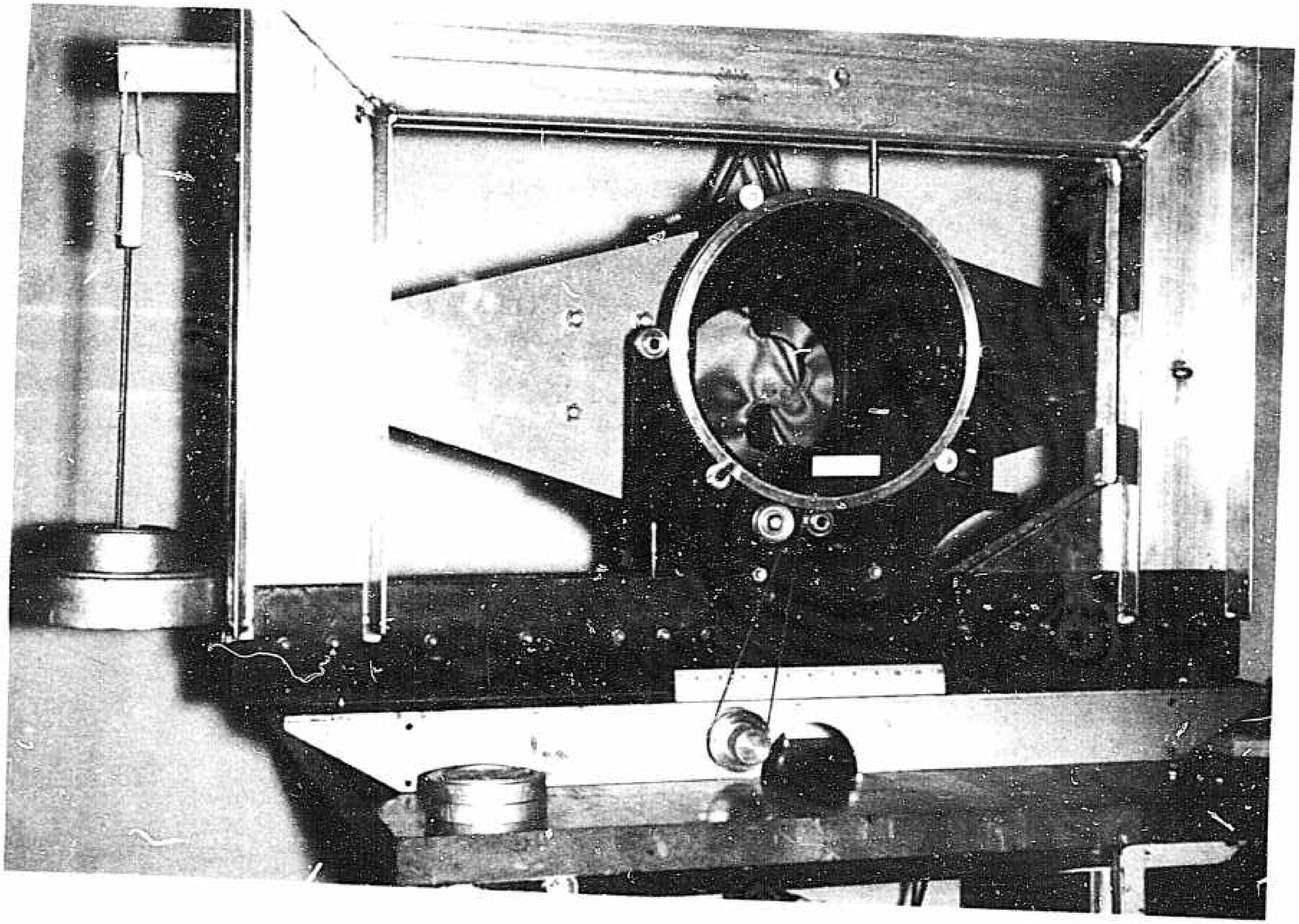


FIGURE NO. 9 PHOTOELASTIC SPECIMENS AND LOAD FRAME

FIGURE NO. 10 PHOTOELASTIC SPECIMENS

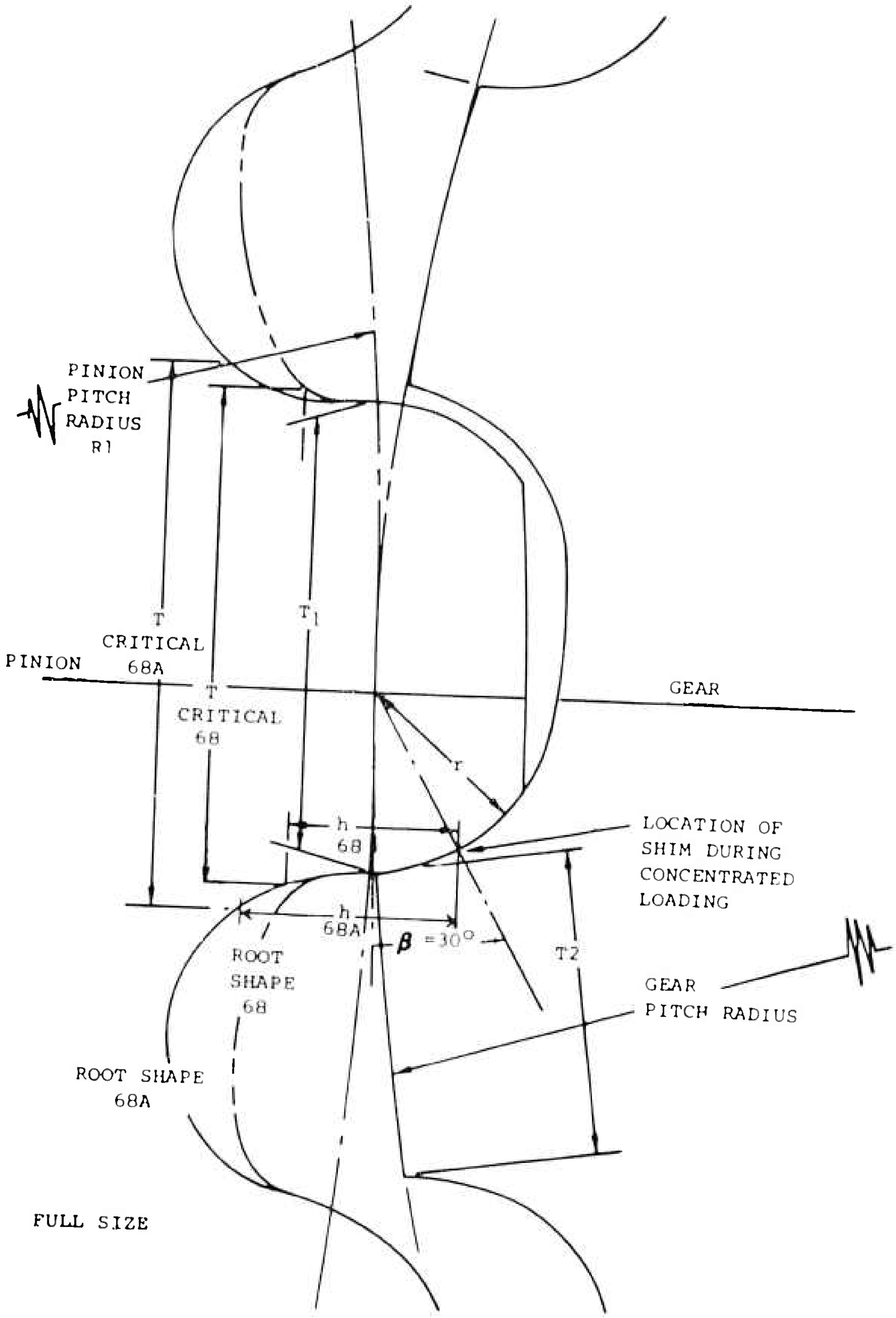


FIGURE NO. 11 GEAR PROFILE AND SYMBOLS

TABLE 2

GEAR-TOOTH CONFIGURATIONS FOR ANALYSIS

Model No.	89	89A	90	93	68	68A
Pitch	5	5	5	5	4.2	4.2
T_1/T_2	1.5	1.5	1.5	1.8	1.5	1.5
T_1/r	1.7	1.7	2.2	1.7	2.6	2.6
Root	Elliptical	Round	Round	Round	Elliptical	Round

Some results of the photoelastic investigation are shown in Figures 12 and 13. Figure 13 shows the expected decrease in the stress concentration factor as fillet radius increases. The experimental results are grouped into two parallel trend lines. This segregation appears as a ratio of bending arm to tooth thickness at the critical section. As tooth thickness and rigidity increase, the stress concentration effect of a constant root radius increases. This is consistent with theory and previous photoelastic examination of gear teeth. Also, in the conformal contact specimens examined, tooth thickness and tooth height were increased together. Although the proportion T_1/r varied, the depth of fillet root required for a smooth transition increased with thickness.

The concentration factor may be considered to be a conversion factor which modifies a simple bending calculation to a realistic solution. The conformal tooth does not resemble the classic cantilever beam in aspect ratio. The beam bending equation assumes this classic ratio, and so does not consider the complex stress field existing in the gear tooth. It must then be modified, in this case by a multiplier of from 1.45 to 2.20.

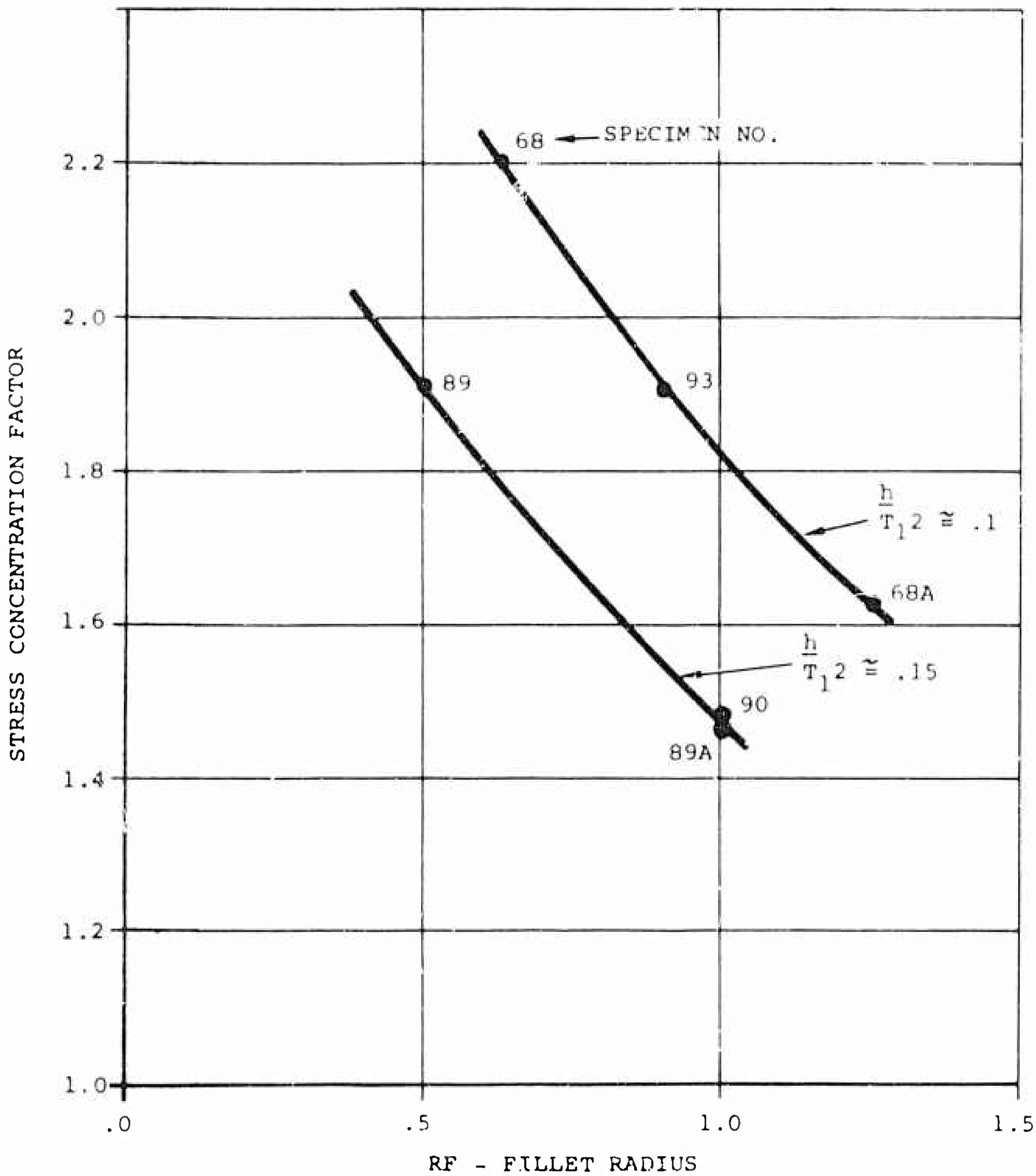


FIGURE NO. 12 CONCENTRATION FACTOR VS FILLET RADIUS

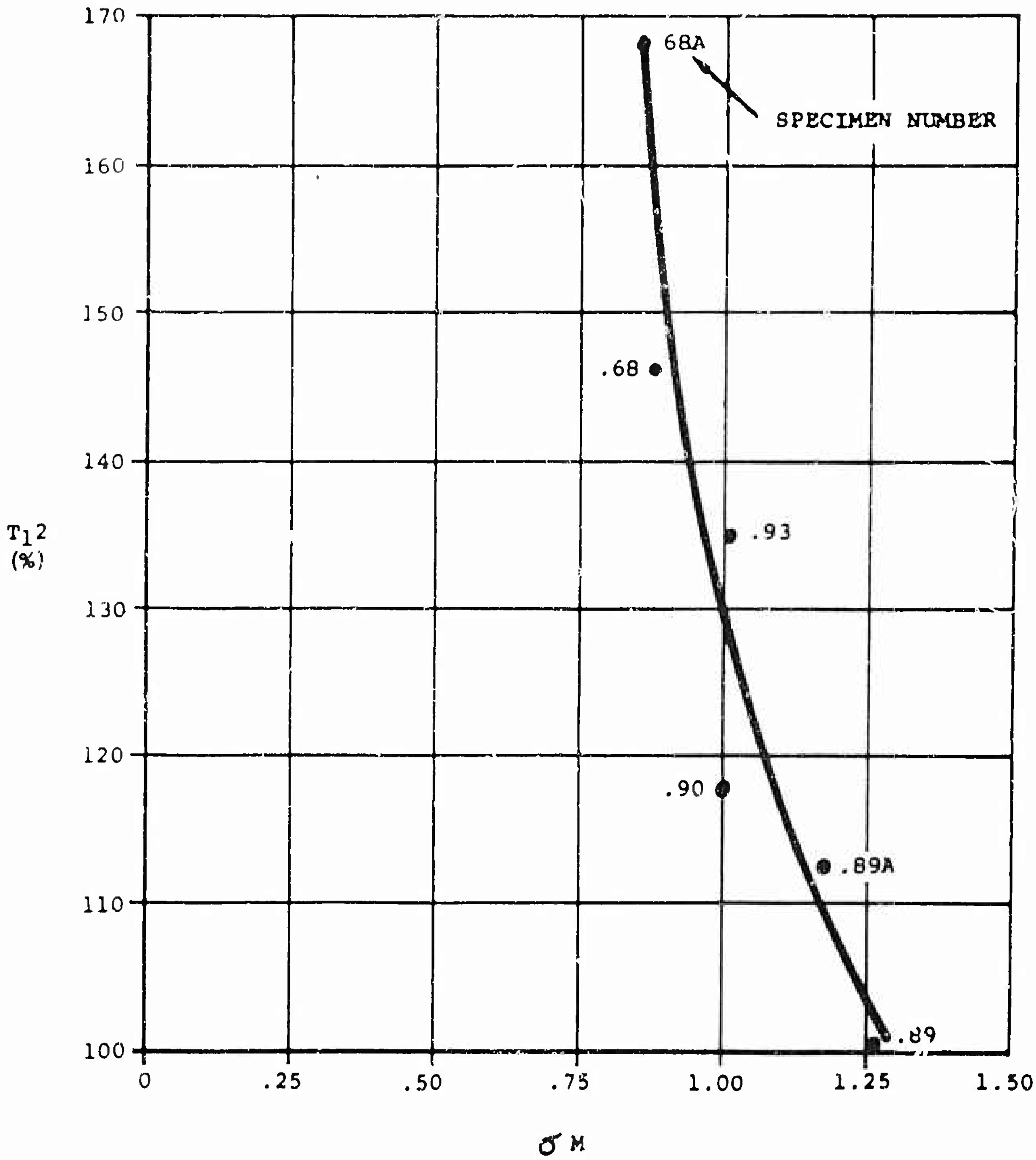


FIGURE NO. 13 TOOTH THICKNESS VS ROOT STRESS

The stress concentration factor is here defined as the ratio of model stress (σ m) to calculated stress (σ c)

σ m = Rf = psi for unit tangential load on a unit face width

Where: R = divisions per unit load

f = psi per division (calibration constant) for unit face width

$$\sigma C = \sigma L + \sigma R = \frac{6h}{T^2} + \frac{\tan \alpha}{T}$$

Where: σ L = bending stress

σ R = radial stress (compressive relief)

h = bending arm

T = tooth thickness at critical section

α = pressure angle

Figure 13 illustrates the effect of increased tooth thickness on tension fillet stress. It is apparent that tooth thickness does not profoundly affect bending stress in the specimens tested. There are at least two explanations: 1) The increased stress concentration factor of the thicker tooth; 2) The increased bending arm of the thicker tooth.

The combination of these two appears to negate the assumed bending strength increase from the thicker tooth.

The stresses in the tension fillets of the concave and convex teeth were compared. In general, the concave tooth fillet stress was one-third higher than the convex. Contrary to predictions, specimen 93 did not show an increased stress differential as compared to those speci-

mens which maintained a 1.5:1 ratio.

Conclusions from Photoelastic Models

1. Increased tooth thickness does not result in proportional decreases in stress. The implication, which remains to be proven by testing, is that the high F/D ratios predicted by analysis may not be required. These ratios, it will be recalled, were predicated on the necessity of a thick tooth for bending strength. Since a thick tooth shows less than the predicted advantage, the most efficient gear design may have a reduced F/D ratio. Another alternate may be to reduce helix angle. Either change would be decidedly beneficial to the practical application of conformal gearing.

2. The full round fillet improves the convex tooth stress field, and it overcomes the effect of increased bending arm.

3. The tension root fillet of the concave gear is generally more critical than the convex gear. This conclusion is supported in part by published reports of tooth breakage occurring first in the concave gear. The solution is apparently not merely to change thickness ratio between concave and convex. This was investigated, although in the direction of further reducing the concave tooth. Continued photoelastic work is being performed to modify the root of the concave tooth and to improve the stress flow in this area. Preliminary indications are that this approach will significantly reduce the stress level.

4. The stress concentration factor used in the analytical calculation (1.5) is acceptable within the expected accuracy of the analysis. It is nonconservative when applied to the

thickest expected teeth; however, it is believed that development of the root fillet will improve the condition.

DESIGN CONSIDERATIONS

Input Conditions

By the requirements of the contract, the area of interest exists between ratios of 20 to 1 to 100 to 1 and horsepower capacities of 250 to 2500. Various power, speed and ratio combinations were studied to develop design solutions within this area. These studies consisted of approximately twenty layouts and associated calculations.

The study conditions chosen are representative of helicopters powered by 250-hp (T-63 class), 1500-hp (T-53 class) and 2500-hp (T-55 class) turbo-shaft engines.

Rotor rpm was taken as generally representative of the helicopter associated with each range of power. The three transmission layouts selected for inclusion in this report were designed to the following requirements:

Config. No.	Rotor rpm	Input rpm	HP per Rotor
SK 13284	350	35,000	250
SK 13283	250	25,000	1,500
SK 13282	200	20,000	2,500

In addition, the lower end of the ratio spectrum was investigated by eliminating the input stage of the 100:1 reduction system. Resultant ratios were between 20 and 25 to 1, typical of drive systems with engine-associated transmissions.

Design Selection

The chosen arrangement of the conformal transmissions is double branch, multistaged. Two or three stages are used

depending upon ratio required. This arrangement was selected to obtain the most satisfactory results from the conformal gear in its present stage of development. As more development is conducted, the optimum arrangement may change. At this time, the considerations of required gear proportions, helix angle, ratio, and reliability make planetary arrangements unattractive. This conclusion is based upon conformal planetary designs for a 3000-horsepower application. The simultaneous requirements of a high F/D ratio, a small-diameter planet gear bearing, and an adequately rigid carrier conflicted to the extent that such a design was not considered feasible.

Number of stages and choice of gear ratio per stage were determined by examining the weight, size, and power loss trade-off. Some considerations were as follows:

1. A lower numerical ratio in the first stage results in a lighter weight system.

2. The gearbox sizes should be comparable to existing planetary transmissions of equivalent capacities. Mounting points should not extend beyond those presently required, to eliminate structural considerations. Very large final-reduction gear diameters are not therefore possible.

3. Further, the thrust of the helical gears must be reacted through the web to the shaft. Extreme gear sizes with heavy loads require impractical thicknesses in web section and shafts to carry the thrust moment and minimize rim deflection.

4. It was concluded from the above that the maxi- practical ratio per stage was approximately 5:1. This resulted in a three-stage configuration for all but the lowest ratio (22.5:1) transmission considered.

The general design advantages of the two-branch, multi-stage system may be listed as follows:

1. Redundancy of the second and third stages by virtue of a dual power path to the rotor shaft. In the event of a gear failure in either stage, this dual power path would insure continued operation. To realize this advantage, provisions for thrust pads must be incorporated on both stages to react the thrust loads resulting from an asymmetric drive condition.

2. A lowered rotor hub is possible since the rotor shaft extends through the transmission. Shaft bearings are located within the housings.

3. In comparison with an equivalent three-stage planetary system, the conformal gear system is heavier in dead weight. This is offset by the reduced power losses which make the conformal gear system lighter in total effective weight. This is discussed and explained under Discussion.

4. The conformal gear system contains fewer major components than an equivalent three-stage planetary system as shown below:

System	No. of Gears	No. of Bearings
Three-stage Planetary	20	23
Three-stage Conformal Gear	14	15

This reduction will tend to reduce the maintenance required. A simplified lubrication system is also foreseen.

Some specific advantages seen in the choice of branches and gear arrangement are as follows:

1. The double-branch, double-gear system reduces individual gear loads by a factor of 4, resulting in smaller pitch diameters.

2. Reduction of torque by 2 for the number 3 and 4 shafts.

3. Elimination of bearing thrust loads by virtue of the opposed helices, for shafts 3, 4, and 5.

4. Reduction of bearing loads by a factor of 2.

Systems with more than two branches were analyzed and discarded for the following reasons:

1. Choice of ratios per stage was restricted.

2. More than two branches does not keep the center clear for the connecting quill shaft.

The problem area most apparent in the study configurations is in adequate support of the pinions. The F/D ratio required by analysis to provide superiority in load capacity results in a wide-face, small-diameter pinion. Shaft size is in some cases insufficient to mount a compatible bearing. Deflections, both bending and torsional, may be above the limits required for a confident prediction of load-carrying ability based upon the full face width. To make the full face width effective, deflections can be, and are, compensated for by the machining of the gear. It is hoped, however, that testing will give evidence that high F/D ratios are not required for conformal gear superiority.

A comparison of the simple deflection equation with the torque capacity/diameter relationship of the conformal

gear indicates that bending deflection is size-sensitive and increases with power input.

1. Diameter D varies as $\overline{\text{Torque}}^{\frac{1}{3}}$. $(T^{\frac{1}{3}})$

2. Deflection varies as Tangential Load, as $\overline{\text{Arm}}^{3}$, and inversely as $\overline{\text{Diameter}}^{4}$.

3. Tangential load varies as Diameter.
Combining, with a constant F/D ratio, in this case = 1:

$$\text{Deflection} = K\frac{WL^3}{D^4}$$

$$L = \frac{D}{2}, \qquad W = \text{Torque} \times \frac{D}{2}$$

$$\text{Substituting:} \quad \delta = \frac{KT}{4D^2}$$

$$\text{However:} \quad D = \sqrt[3]{\frac{T}{K'}} \quad \text{(From Design Curve, Figure 5)}$$

$$\text{Therefore:} \quad \delta = K''\frac{T}{T^{2/3}} = K''\,T^{.33}$$

For the assumptions of constant F/D ratio, a bending arm proportionate to gear diameter, and proportionate shaft wall thickness, it can be seen that deflection varies as a power of torque. That is, a conformal pinion designed to transmit double the torque of another by this analysis will encounter 25% more deflection.

Input shaft thrust bearings present a problem in the configurations studied. The small shafts, high (20,000-35,000) rpm, and considerable forces involved severely limit the design life of rolling element bearings. The immediate solution proposed would use involute spur gears for this stage, thereby eliminating

thrust load. Weight increase for this change would be small since the first-stage gears constitute about 7% of the total transmission weight. A more effective long-term solution would utilize hydrodynamic bearings, not only in the first stage, but throughout the transmission. It is believed that satisfactory development of these bearings would contribute greatly to the weight advantage to be gained from higher-capacity gearing. Decreased gear size places proportionately more load upon bearings which increasingly influence the weight and size of the resulting transmission.

Initial layouts of the conformal transmission system located the input pinion in the center of the gear case (SK 13307, see Figure 24). Despite the compactness of this arrangement, it was discovered that this design both lengthened the second-stage pinion shaft and restricted its diameter. The result was that bending deflections became critical. This arrangement was therefore rejected for the power range under consideration. The causes for rejection were overcome by relocating the input pinion and gear at the bottom of the transmission and driving upwards through a splined shaft. The overall depth of the gearbox was necessarily increased.

With the exception of the first stage, all bearings have been located in two planes in the main case. This considerably simplifies the design of the case to accept the bearing reactions. It also eases the problems of assembly and manufacture. The rotor shaft is axially constrained through a thrust bearing; this bearing may be sized to accept rotor loads. The remaining gears of the second and third stages are allowed to float axially and are positioned by their opposed helices. This is a necessary requirement with the herringbone gear; the apexes of the mating helices must be free to align and thus equalize tooth load.

To equalize load between branches, the pinion-to-gear relationship must be closely maintained. This situation is common to compound planetaries as well, and has been successfully met in practice. Floating the central gear has been used in a certain heavy-duty automotive transmission to provide load equalization. In this application, the two branches were diametrically opposed so that all loads were equal and opposite. This method, while not directly applicable to the study layouts, would be considered in the future.

DISCUSSION

The results of the weight study made from layouts and calculation are shown in Figures 14 through 20. The conformal-gear two- and three-stage transmissions are compared to conventional involute planetary systems of the same number of stages and under the same conditions. The planetary weights are derived by the method of R. J. Willis (see Bibliography) and checked, where possible, by actual weights. All actual planetary points are in the low-ratio area of 17 to 20 to 1.

Two weights are calculated: dead weight and total effective weight. The dead weight is simply the calculated weight of the material. Total effective weight includes the weight equivalent of the transmission power loss as well. It is considered a better means of comparing various systems. It recognizes that inefficiencies represent additional power required, which could be lifting additional weight if it were not diverted into friction and windage, and lost. As an added penalty, such losses result in larger oil cooling systems, pumps, sumps, and lines.

Total Effective Weight = TEF

TEF = Dead Weight + (% Power Loss x hp x lb/hp)

$$\text{lb/hp} = \frac{\text{helicopter gross weight}}{\text{total installed horsepower}} = 7.5$$

The power loss per single conformal mesh has been assumed at .75% of transmitted power. The loss per planetary stage (2 meshes) has been assumed at 1.5 times as much, or 1.12%. The planet power loss per mesh is somewhat lower than a nonplanetary equivalent because in a planetary potential power is less than transmitted power. (Reference: Manual of Gear Design, by Earle Buckingham, and other standard works on planetary design.) The con-

formal mesh loss of .75% is considered to be conservatively high; the literature describes the conformal gear as more efficient than the involute because sliding is reduced. If this is verified in aircraft type ground gearing, the relative standing of conformal versus involute systems will improve when total effective weight is used for comparison.

The dead-weight comparison (Figure 14) shows the conformal transmission system as 15 to 20% heavier than an equivalent planetary in the higher horsepowers.

The total effective weight comparison (Figure 15) shows the conformal transmission as somewhat lighter (5 to 20%). This reversal of position is due to the lower power loss calculated for a conformal transmission with number of stages equivalent to the planetary.

In both transmission types, the maximum ratio per stage has been set as 5:1. The three-stage planetary systems used for comparison have a star (non-orbiting) first stage, since the high input speeds would induce prohibitive centrifugal forces in orbiting planets.

Figure 16 compares dead weight to output shaft torque for various ratios. Figure 17 compares total effective weight to output shaft torque. Figures 18 and 19 compare both weights to ratio for various horsepowers. The relative positions of conformal and involute planetary gears are the same as in the previous plots.

The volume of the study transmissions has been plotted in Figure 20 for two ratios: 101:1, and 22.5:1.

Cost of prototype transmissions has not been charted, since the development of the manufacturing procedure, in particular the inspection of the finished gear form, is susceptible to wide variations in cost. At the conclusion of the proposed Phase II, which includes the manufacture of test gears, prototype transmission costs

DISCUSSION

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Two weights are calculated: dead weight and total effective weight. The dead weight is simply the calculated weight of the material. Total effective weight includes the weight equivalent of the transmission power loss as well. It is considered a better means of comparing various systems. It recognizes that inefficiencies represent additional power required, which could be lifting additional weight if it were not diverted into friction and windage, and lost. As an added penalty, such losses result in larger oil cooling systems, pumps, sumps, and lines.

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TEF = Dead Weight + (% Power Loss x hp x lb/hp)

$$\text{lb/hp} = \frac{\text{helicopter gross weight}}{\text{total installed horsepower}} = 7.5$$

The power loss per single conformal mesh has been assumed at .75% of transmitted power. The loss per planetary stage (2 meshes) has been assumed at 1.5 times as much, or 1.12%. The planet power loss per mesh is somewhat lower than a nonplanetary equivalent because in a planetary potential power is less than transmitted power. (Reference: Manual of Gear Design, by Earle Buckingham, and other standard works on planetary design.) The con-

may be evaluated with more confidence. There seems to be no inherent reason why the conformal gear, with developed tooling, should be more expensive than the involute. It also appears that much of the involute gear technology, such as cutting and grinding machinery and heat treatment, will adapt to the conformal gear. For example, the Circarc gear is hobbed on conventional machines, and helicalform grinding machines such as the National Broach Red Ring SGF-12 are available for finish grinding.

It is suggested that the trend curves presented in this section be considered as tentative conclusions only. At the completion of the proposed testing phase, these trends will be reviewed and corrected. At that time, a more confident conclusion can be made as to the weight and cost merits of the conformal transmission for V/STOL service.

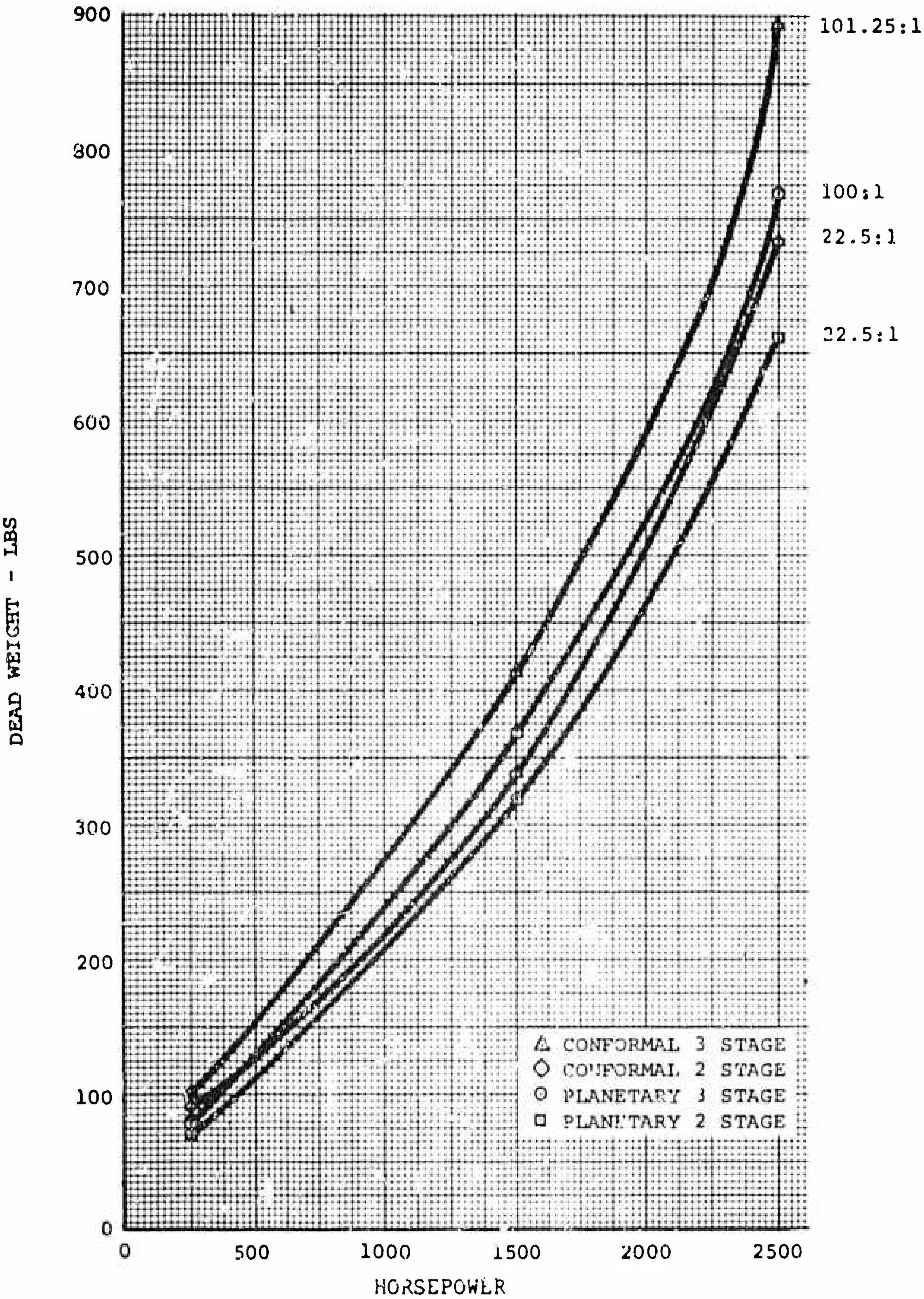

FIGURE NO. 14 DEAD WEIGHT VS HORSEPOWER

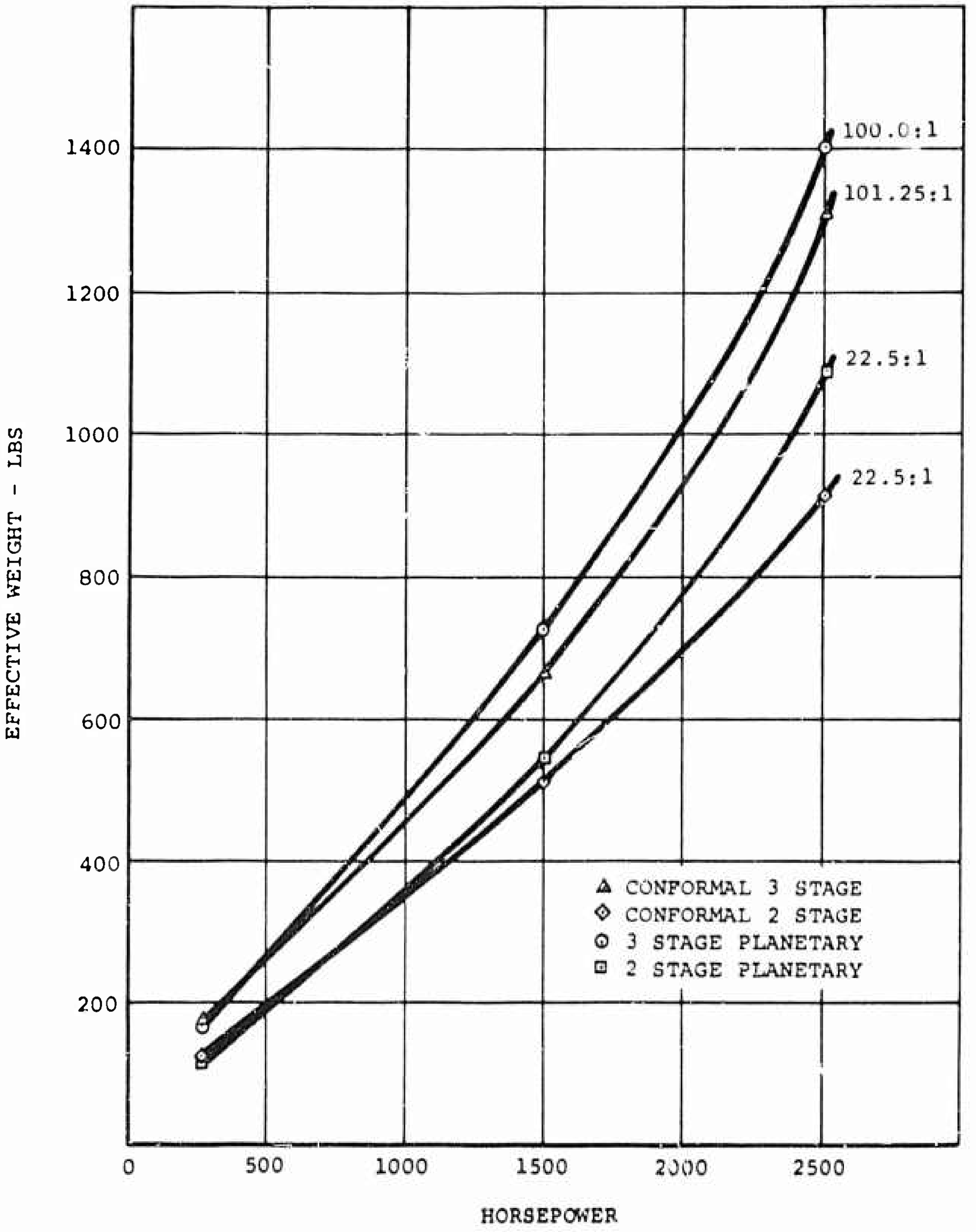

FIGURE NO. 15 EFFECTIVE WEIGHT VS HORSEPOWER

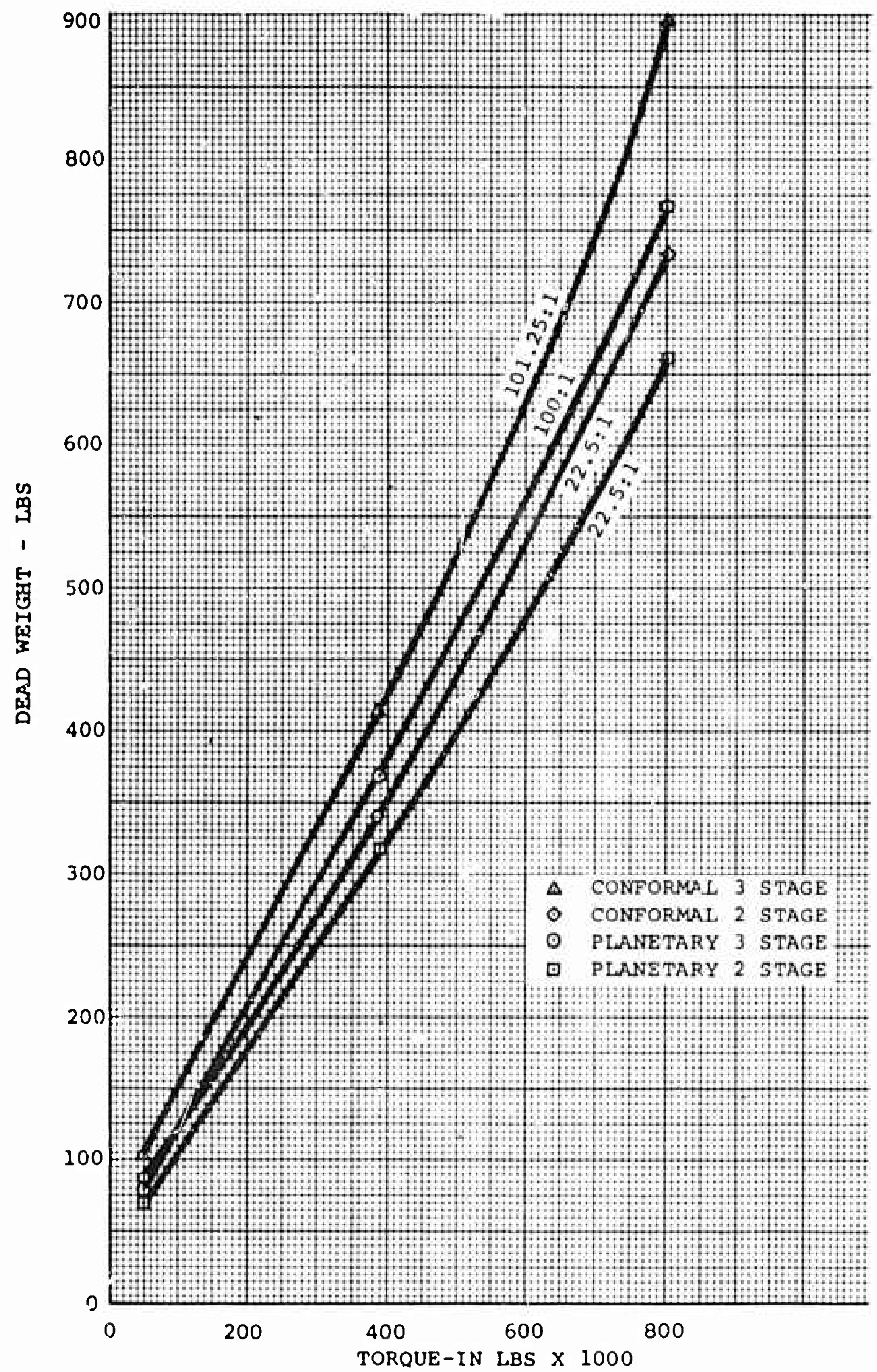

FIGURE NO. 16 DEAD WEIGHT VS TORQUE

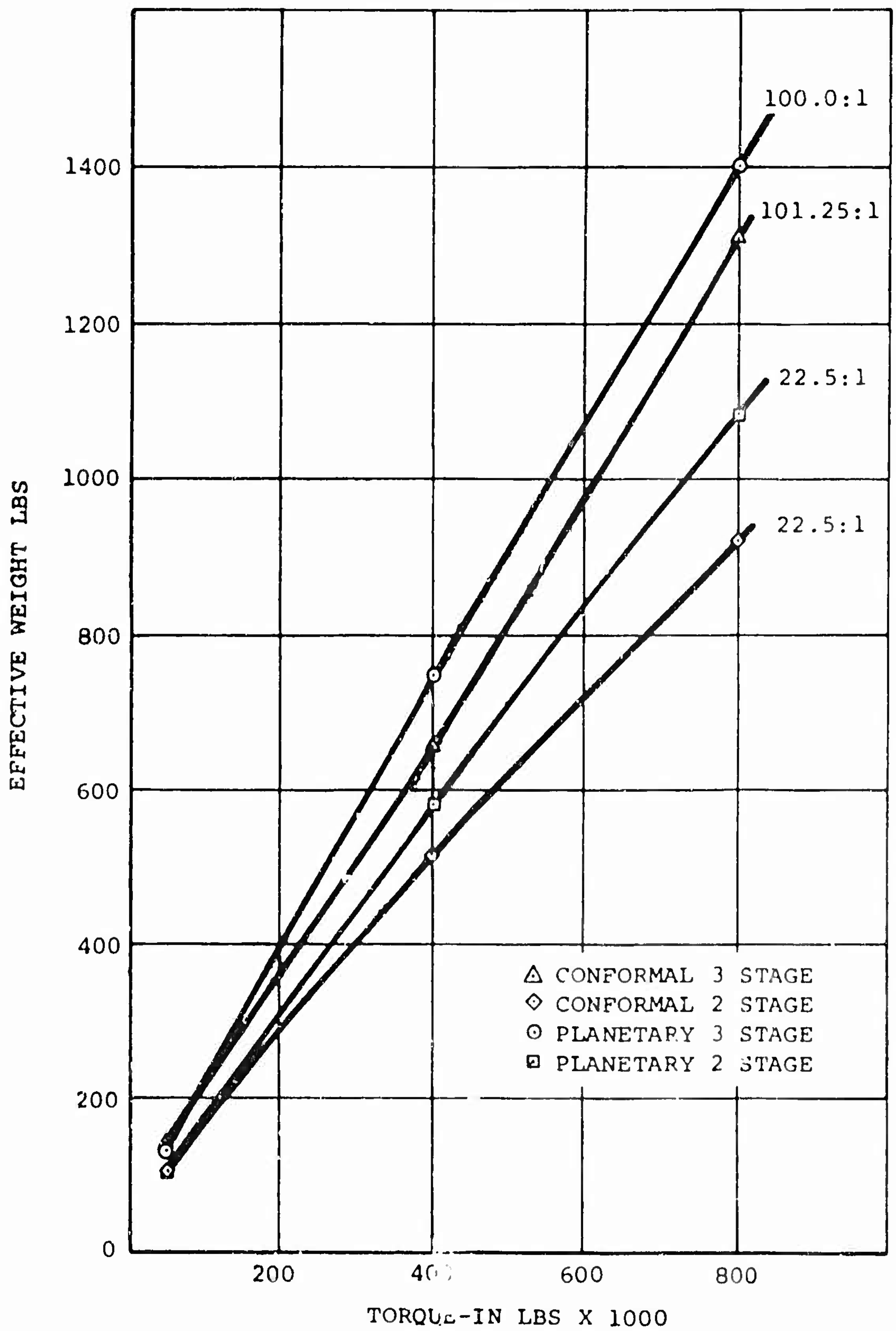

FIGURE NO. 17 EFFECTIVE WEIGHT VS TORQUE

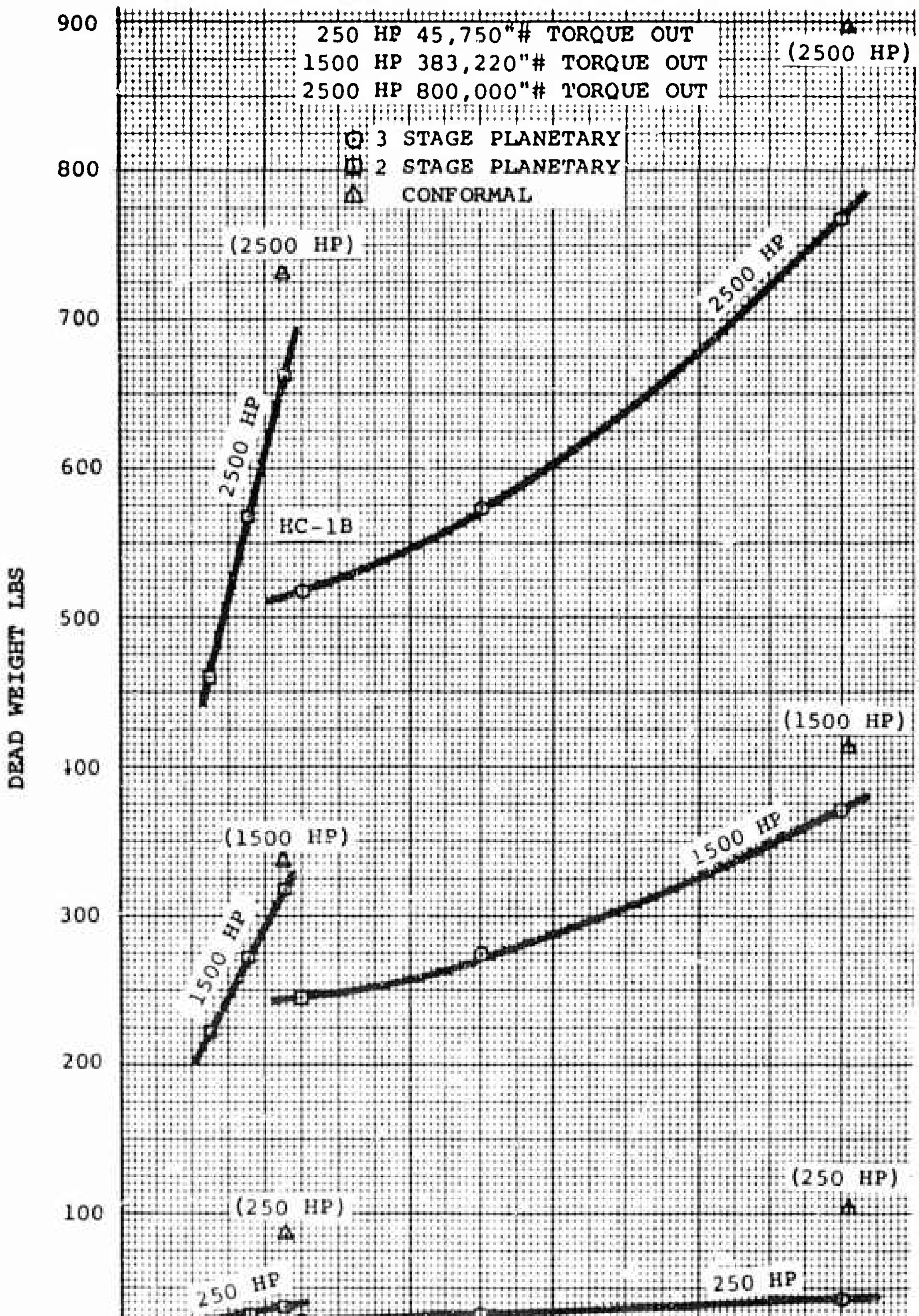

FIGURE NO. 18 DEAD WEIGHT VS RATIO

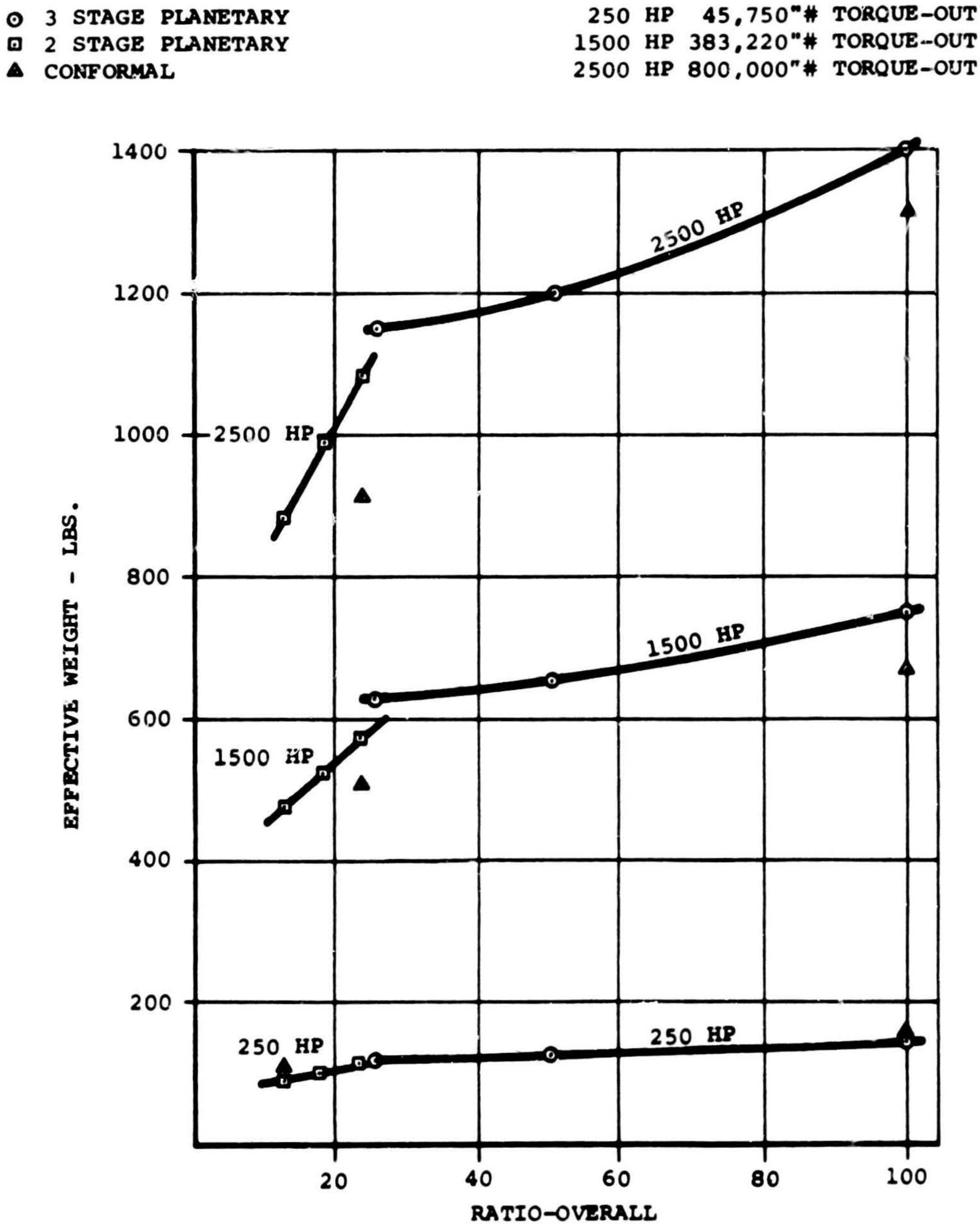

FIGURE NO. 19 EFFECTIVE WEIGHT VS RATIO

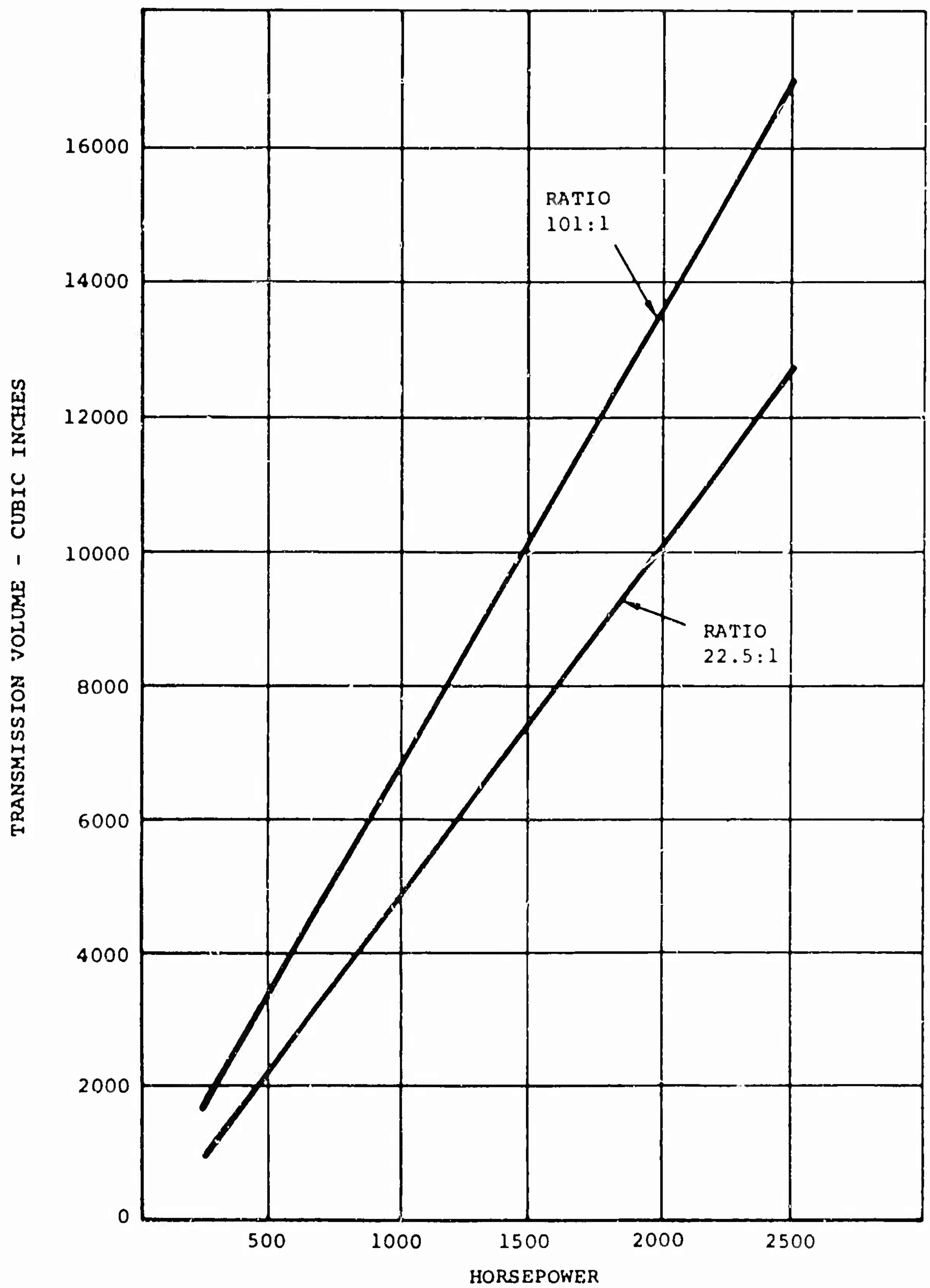

FIGURE NO. 20 VOLUME VS HORSEPOWER

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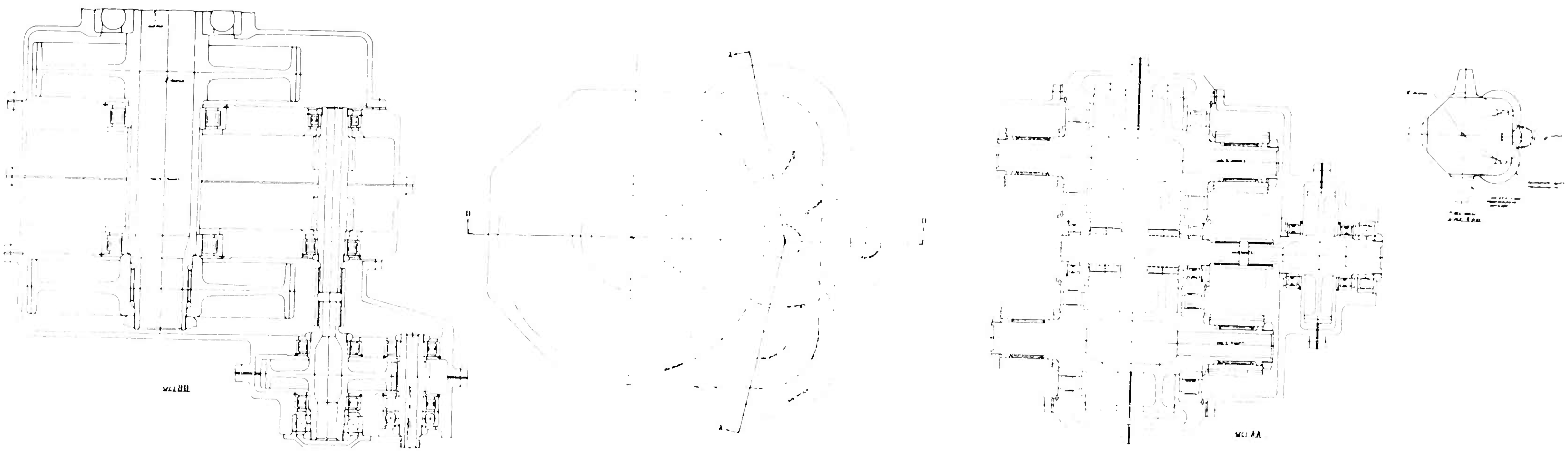


FIGURE NO. 21. CONFORMAL GEAR TRANSMISSION (SK 13282)

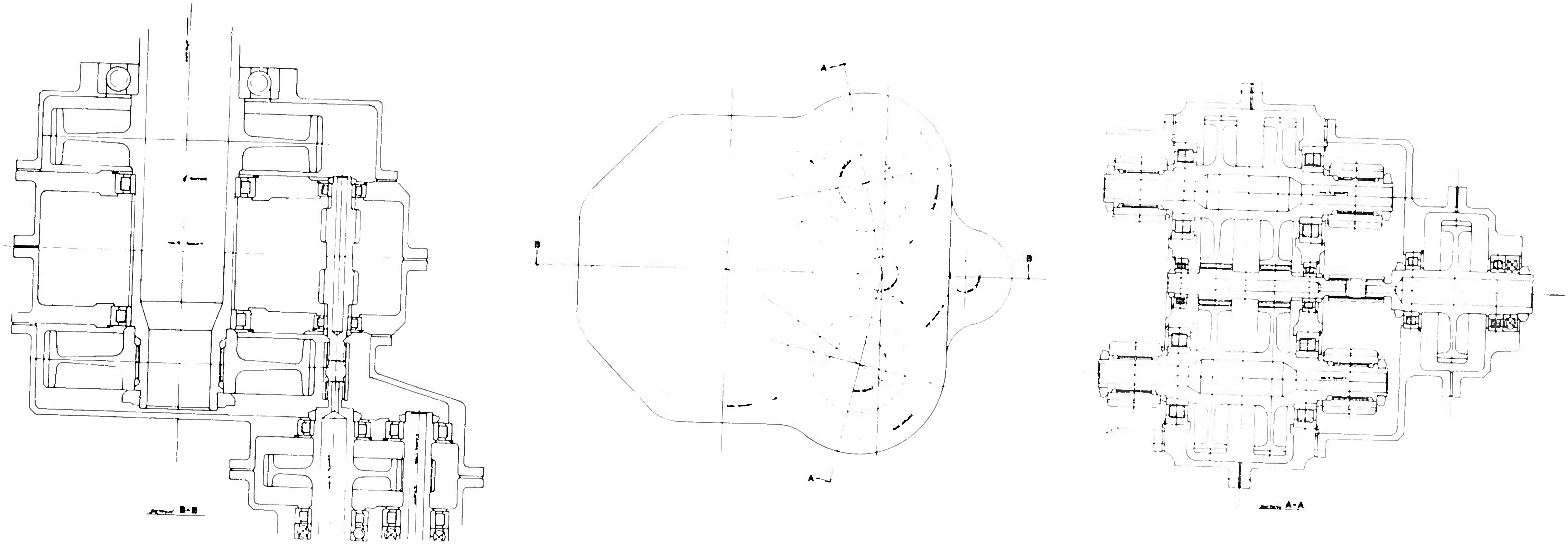


FIGURE NO. 22. CONFORMAL GEAR TRANSMISSION (SK 13283)

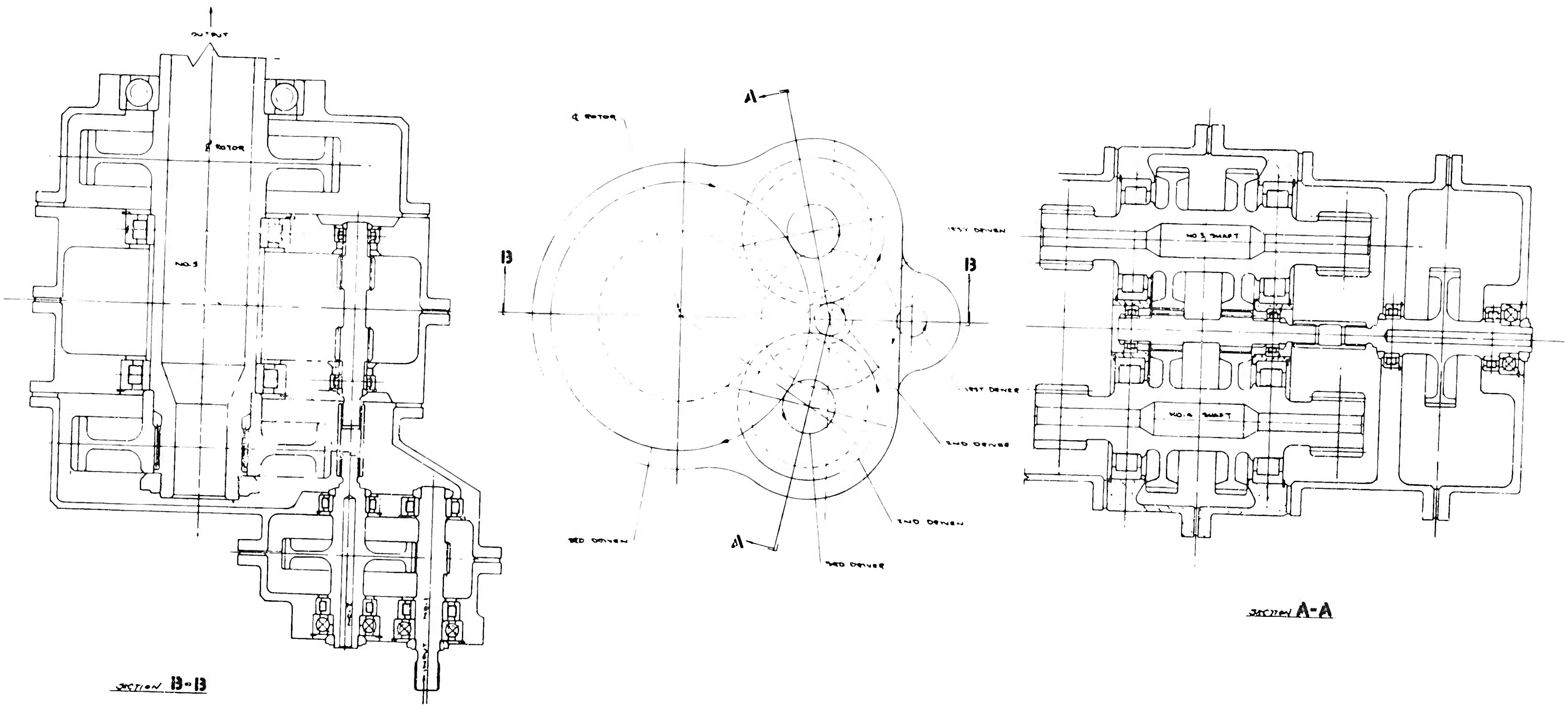


FIGURE NO. 23. CONFORMAL GEAR TRANSMISSION (SK 13284)

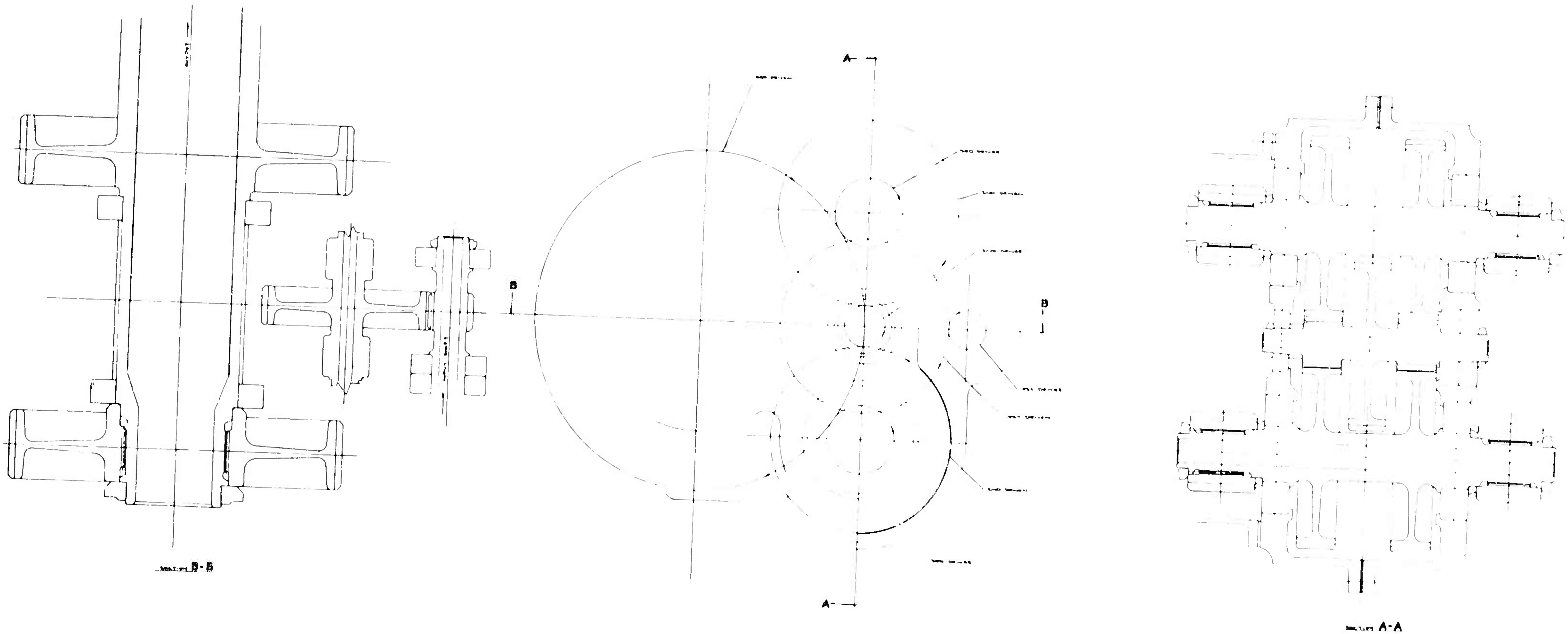


FIGURE NO. 24. CONFORMAL GEAR TRANSMISSION (SK 13307)

APPENDIX B - STRESS ANALYSIS

CONFORMAL CONTACT GEAR TRANSMISSION
BEARING LOADS AND REACTIONS

Horse Power	2,500
Speed	20,000 R.P.M.
Overall Ratio	101.25 to 1
First Stage Ratio	4.5 to 1
Second Stage Ratio	4.5 to 1
Third Stage Ratio	5.00 to 1

$$\text{Torque} = \frac{\text{HP} \times 63{,}025}{n} = \frac{2{,}500 \times 63{,}025}{20{,}000}$$

Torque = 7,900 in.lb.

FIRST STAGE

Torque = 7,900 in.lb.

Face Width	=	2.20 in.
(8T) Pitch R. Driver	=	1.10 in.
(36T) Pitch R. Driven	=	4.95 in.

SECOND STAGE

Torque = 7,900 x 4.5 = 35,580

$$\frac{35{,}580}{4} = 8{,}990 \text{ in. lb./mesh}$$

$$R_1 = \sqrt[3]{\frac{\text{Torque}}{4800}} = \sqrt[3]{\frac{8{,}990}{4800}}$$

$$R_1 = \sqrt[3]{1.851} = 1.228$$

Face Width	=	2.456 in.
(8T) Pitch R. Driver	=	1.228 in.
(36T) Pitch R. Driven	=	5.526 in.

THIRD STAGE

Torque = 35,580 x 4.5

= 160,000 in.lb.

$\frac{160,000}{4}$ = 40,000 in.lb./mesh

Face Width = 4.054 in.

(8T) Pitch R. (Driver) = 2.027 in.

(40T) Pitch R. (Driven) = 10.135 in.

$$R_1 = \sqrt[3]{\frac{40,000}{4,800}} = \sqrt[3]{8.325}$$

R_1 = 2.027

BEARING LOADS - FIRST STAGE

$$\text{Tang. F} = \frac{2T}{D_p} = \frac{2 \times 7900}{2.20} = 7182 \text{ lb.}$$

Sep. S = F x Tan Press. $\angle$ = 7182 x .577 = 4150 lb.

Use .75 as a Cubic Mean Load for All Bearing Load Calculations

Tang. F = 7182 x .75 = 5386 lb.

Sep. S = 4150 x .75 = 3100 lb.

BEARING LOADS - SECOND STAGE (DOUBLE MESH)

$$\text{Ratio} = \frac{\text{Pitch R (First Stage Driven)}}{\text{Pitch R (Second Stage Driver)}} \quad \frac{4.95}{1.228} = 4.03$$

Tang. F = 5386 x 4.03 = 21,700 lb.

$\frac{21,700 \text{ lb.}}{4}$ = 5420 lb./mesh

$$\text{Sep. S} = \frac{\text{F x Tan Pres } \angle}{\text{Cos Helix } \angle} = \frac{21,700 \times .577}{.905} = 13,850 \text{ lb.}$$

$\frac{13,850}{4}$ = 3,460 lb./mesh

BEARING LOADS - SECOND STAGE (CONT.)

Thrust T = F x Tan Helix $\measuredangle$ = 21,700 x .471 = 10,220

$\frac{10,220}{4}$ = 2560 lb./mesh

BEARING LOADS - THIRD STAGE (DOUBLE MESH)

Ratio = $\frac{\text{Pitch R (Second Stage Driven)}}{\text{Pitch R (Third Stage Driver)}}$ $\frac{5.526}{2.027}$ = 2.72

F = 2.72 x 21,700 = 59,000 lb.

$\frac{59,000}{4}$ = 14,750 lb./mesh

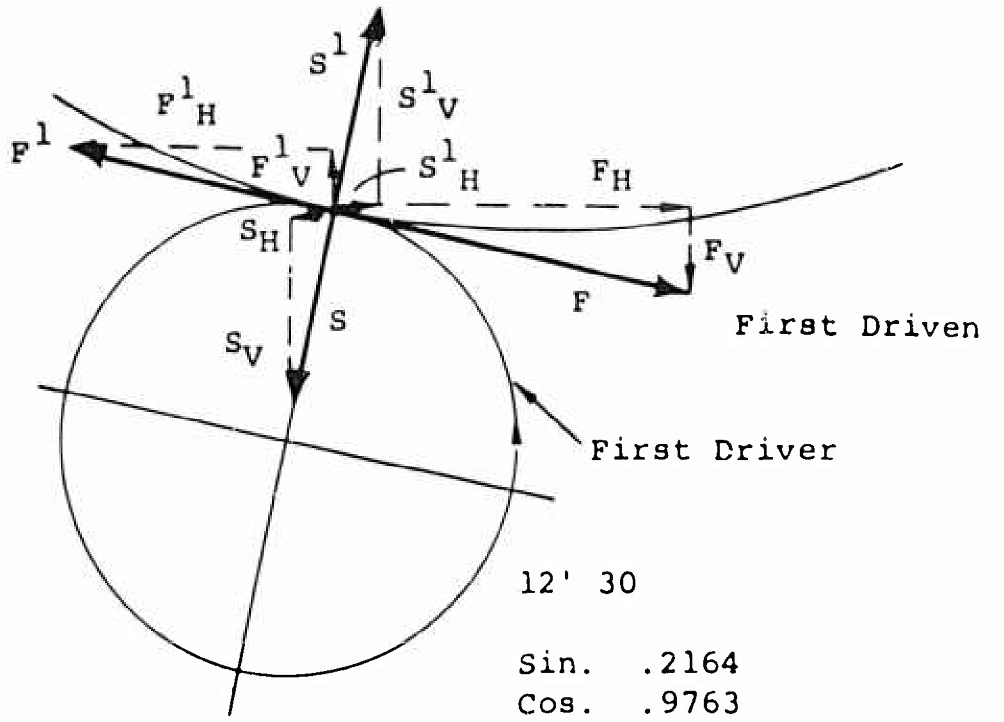
S = 2.72 x 13,850 = 37,700

$\frac{37,700}{4}$ = 9,420 lb./mesh

T = 2.72 x 10,220 = 27,880

$\frac{27,880}{4}$ = 6,960 lb./mesh

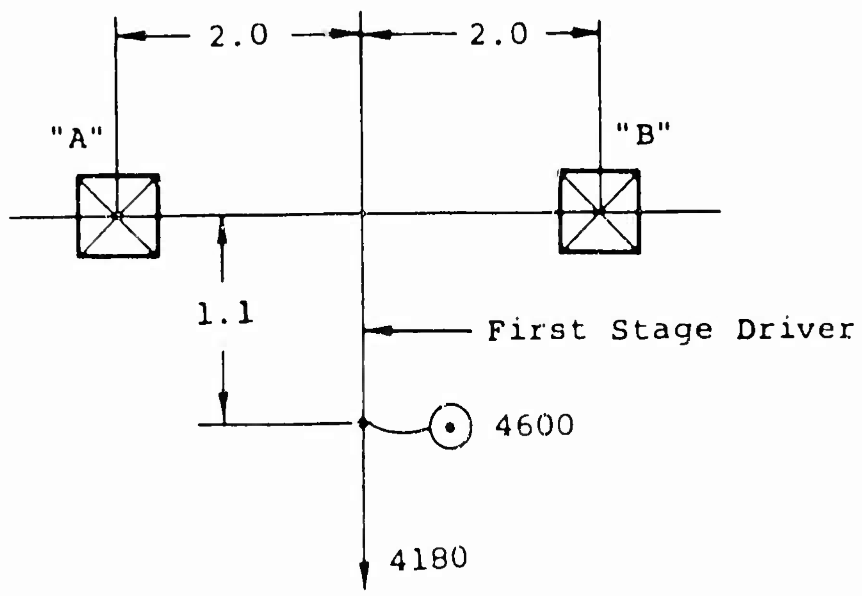
FORCE RESOLUTION - FIRST STAGE *



Sin. .2164
Cos. .9763
Tan. .2217

F_H = 5386 x .976 = 5270 lb.
F_V = 5386 x .216 = 1165 lb.
S_H = 3100 x .216 = 670 lb.
S_V = 3100 x .976 = 3015 lb.

V = 1165 + 3015 = 4180 ↓
H = 5270 - 670 = 4600 ⊙



* NOTE: First stage mesh on layouts SK13282, SK13283, SK13284 is approximately 90° c'clockwise from the position in this analysis. This will not alter the bearing loads as determined by this analysis.

FIRST STAGE DRIVER BEARING LOADS

$\sum M @ A_x = 0$

$= +4180 \times 2.0 - RB_x (4.0)$

$= 8360 - 4 RB_x$

$RB_x = \frac{8360}{4} = 2090$ lb.

$\sum M @ A_y = 0$

$= -4600 \times 2.0 + RB_y (4.0)$

$= -9200 + 4 RB_y$

$RB_y = \frac{9200}{4} = 2300$

$\sum RB = \sqrt{\overline{RB_x}^2 + \overline{RB_y}^2} = \sqrt{(2090)^2 + (2300)^2}$

$= \sqrt{4,360,000 + 5,280,000}$

$= \sqrt{9,640,000}$

$= 3,110$ lb.

DYNAMIC CAPACITY REQUIRED (SPEED 20,000 R.P.M.)

$C/P = \left(\frac{LH \times n}{500 \times 33\ 1/3}\right)^{3/10} = \left(\frac{1,000 \times 20,000}{500 \times 33\ 1/3}\right)^{3/10}$

$= (1200)^{3/10}$

$= 8.39$

Brg. "B" $= 3110 \times 8.39$

$= 26,060$ lb.

FIRST STAGE DRIVER BEARING LOADS (CONT.)

ΣM @ $B_x = 0$

$= -4180 \times 2.0 + RA_x\ (4.0)$

$= -8360 + 4\ RA_x$

$RA_x = \frac{8360}{4} = 2090$ lb.

ΣM @ $B_y = 0$

$= -4600 \times 2.0 + RA_y\ (4.0)$

$= -9200 + 4\ RA_y$

$RA_y = \frac{9200}{4} = 2300$ lb.

$\Sigma RA = \sqrt{\overline{RA_y}^2 + \overline{RA_y}^2} = \sqrt{(2090)^2 + (2300)^2}$

$= \sqrt{4{,}360{,}000 + 5{,}280{,}000}$

$= \sqrt{9{,}640{,}000}$

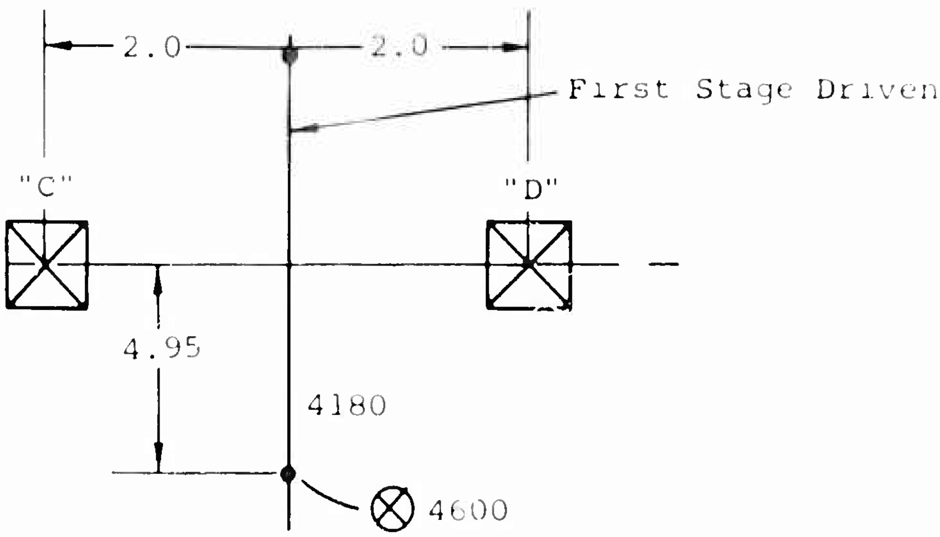
$RA = 3{,}110$ lb.

Dynamic Capacity Required (Brg "A")

$C/P = 8.39$

$8.39 \times 3110 = 26{,}060$ lb.

FIRST STAGE DRIVEN BEARING LOADS



ΣH = 5270 - 670   4600 $\otimes$

ΣV = 3015 + 1165 = 4180 $\uparrow$

Load at Bearing "C" = 3110 lb.

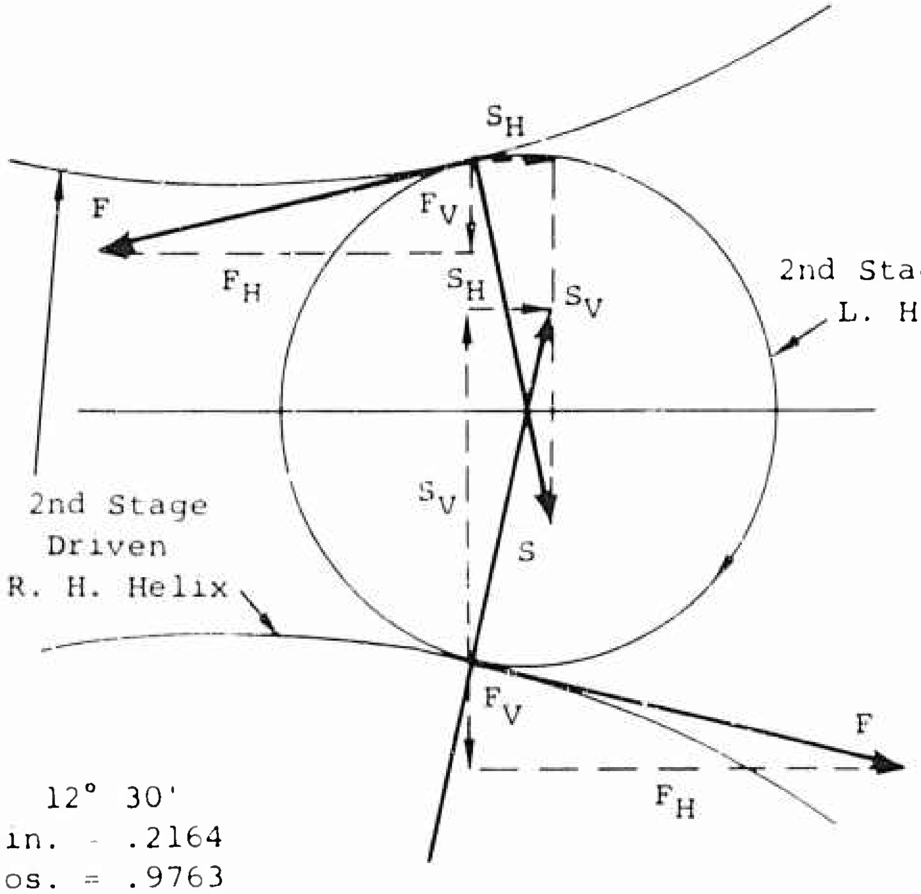
Load at Bearing "D" = 3110 lb.

DYNAMIC CAPACITY REQUIRED (SHAFT SPEED 4450)

C/P 5.5

5.5 x 3110 = <u>17,120 lb.</u>

FORCE RESOLUTION - SECOND STAGE DRIVER



12° 30'
Sin. = .2164
Cos. = .9763

F_H = 5420 x .976 = 5300 lb.

F_V = 5420 x .216 = 1172 lb.

S_H = 3460 x .216 = 748 lb.

S_V = 3460 x .976 = 3380 lb.

NOTE: Thrust loads are equal and opposite by inspection and are therefore disregarded in these calculations.

BOTTOM MESH

Σ V = 3380 - 1172 = 2208 ↑

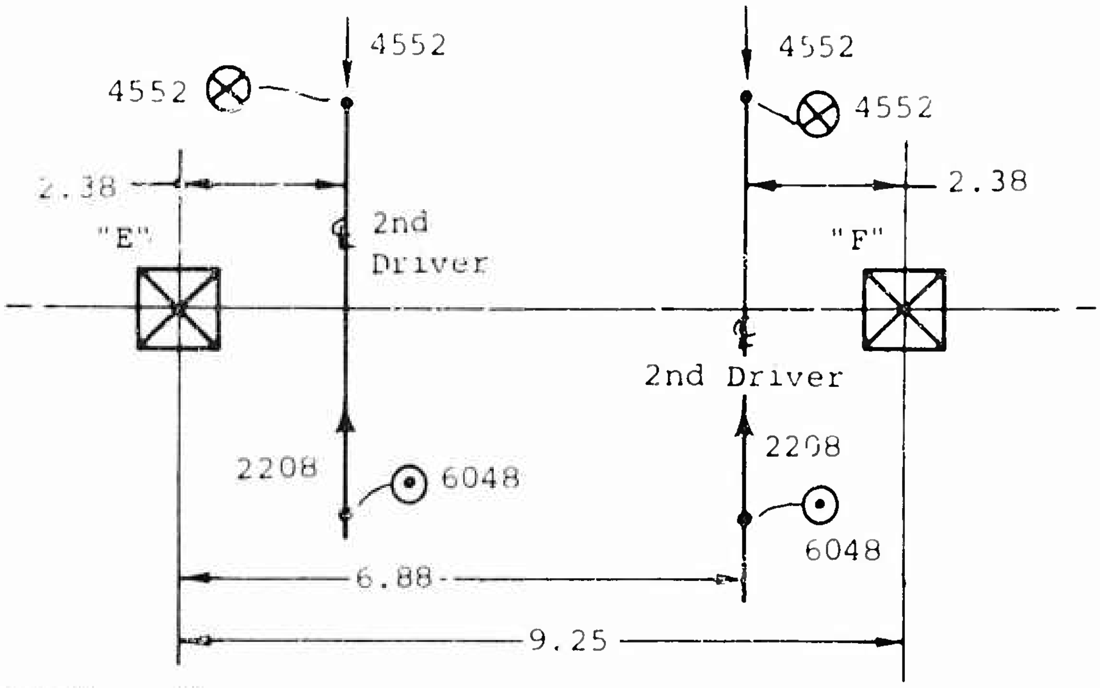
Σ H = 5300 + 748 = 6048 ⊙

UPPER MESH

Σ V = 1172 + 3380 = 4552 ↓

Σ H = 5300 - 748 = 4552 ⊗

2ND STAGE DRIVER BEARING LOADS



$\Sigma M @ E_x = 0$

$= +4552(2.38) + 4552(6.88) - 2208(2.38) - 2208(6.88) - RF_x(9.25)$

$= +10,840 + 31,400 - 5,270 - 15,200 - 9.25\ RF_x$

$RF_x = \frac{21,770}{9.25} = 2,353$ lb.

$\Sigma M @ E_y = 0$

$= +4552(2.38) + 4552(6.88) - 6048(2.38) - 6048(6.88) + RF_y(9.25)$

$= +10,840 + 31,400 - 14,400 - 41,600 + 9.25\ RF_y$

$RF_y = \frac{13,760}{9.25} = 1,489$ lb.

$\Sigma RF = \sqrt{\overline{RF_x}^2 + \overline{RF_y}^2} = \sqrt{(2,353)^2 + (1,489)^2}$

$= \sqrt{5,510,000 + 2,220,000}$

SECOND STAGE DRIVER BEARING LOADS (CONT.)

$\Sigma RF = \sqrt{7{,}730{,}000}$

$RF = 2{,}784$ lb.

$\Sigma M @ F_x = 0$

$= +2208\ (2.38) + 2208\ (6.88) - 4552\ (2.38) - 4552\ (6.88) + RE_x\ (9.25)$

$= +5{,}270 + 15{,}200 - 10{,}840 - 31{,}400 + 9.25\ RE_x$

$RE_x = \dfrac{21{,}770}{9.25} = 2{,}353$ lb.

$\Sigma M @ F_y = 0$

$= +6048\ (2.38) + 6048\ (6.88) - 4552\ (2.38) - 4552\ (6.88) - RE_y\ (9.25)$

$= +14{,}400 + 41{,}600 - 10{,}840 - 31{,}400 - 9.25\ RE_y$

$RE_y = \dfrac{13{,}760}{9.25} = 1{,}489$ lb.

$\Sigma RE = \sqrt{\overline{RA_x}^2 + \overline{RA_y}^2} = \sqrt{(2{,}353)^2 + (1{,}489)^2}$

$= \sqrt{5{,}510{,}000 + 2{,}220{,}000} = \sqrt{7{,}730{,}000}$

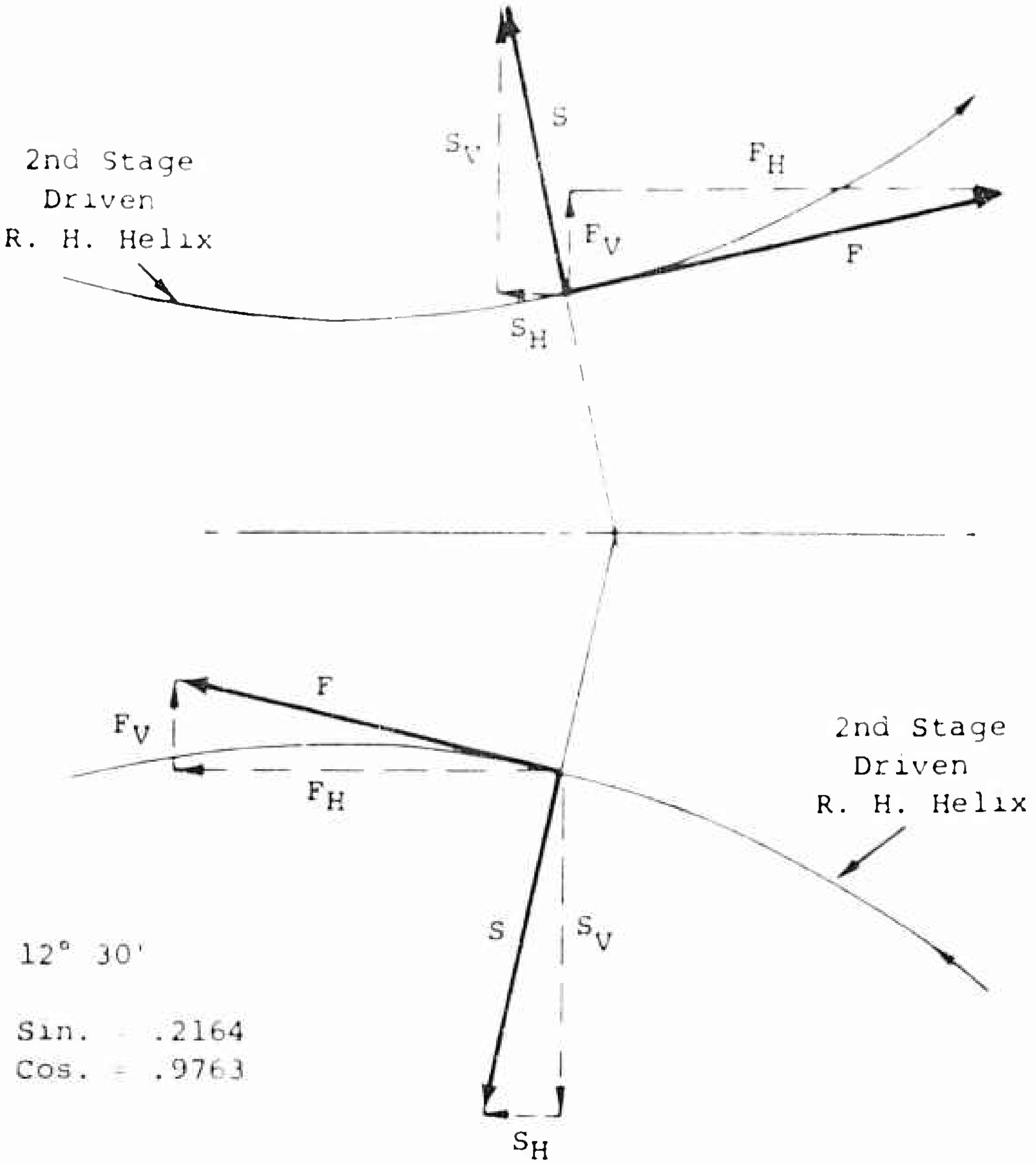
<u>$RE = 2{,}784$ lb.</u>

Dynamic Capacity Required (Shaft Speed 4450 R.P.M.)

$C/P = 5.5$

C for brgs. "E" & "F" = 5.5 x 2784 = 15,320 lb.

FORCE RESOLUTION - SECOND STAGE DRIVEN



12° 30'

Sin. = .2164
Cos. = .9763

F_H = 5420 x .976 = 5300 lb.
F_V = 5420 x .216 = 1172 lb.
S_H = 3460 x .216 = 748 lb.
S_V = 3460 x .976 = 3380 lb.

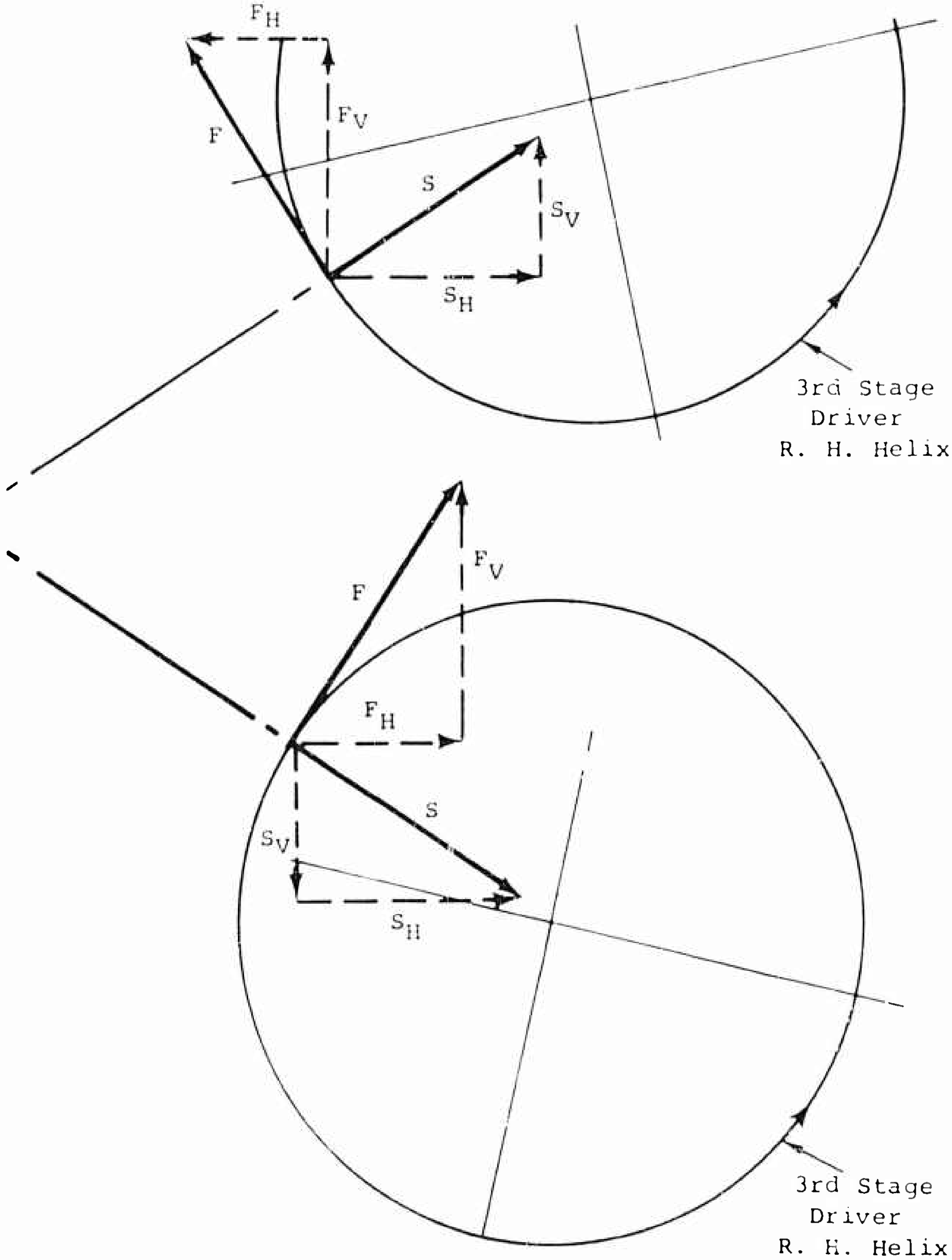
BOTTOM MESH

ΣV = 3380 - 1172 = 2208
ΣH = 5300 + 748 = 6048

TOP MESH

ΣV = 3380 + 1172 = 4552
ΣH = 5300 - 748 = 4552

FORCE RESOLUTION - THIRD STAGE DRIVER



FORCE RESOLUTION - THIRD STAGE DRIVER (CONT.)

$32°30'$

Sin = .537

Cos = .843

F_H = 14,750 x .537 = 7,920 lb.

F_V = 14,750 x .843 = 12,420 lb.

S_H = 9,420 x .843 = 7,930 lb.

S_V = 9,420 x .537 = 5,060 lb.

BOTTOM MESH

ΣV = 12,420 - 5,060 = 7,360 $\uparrow$

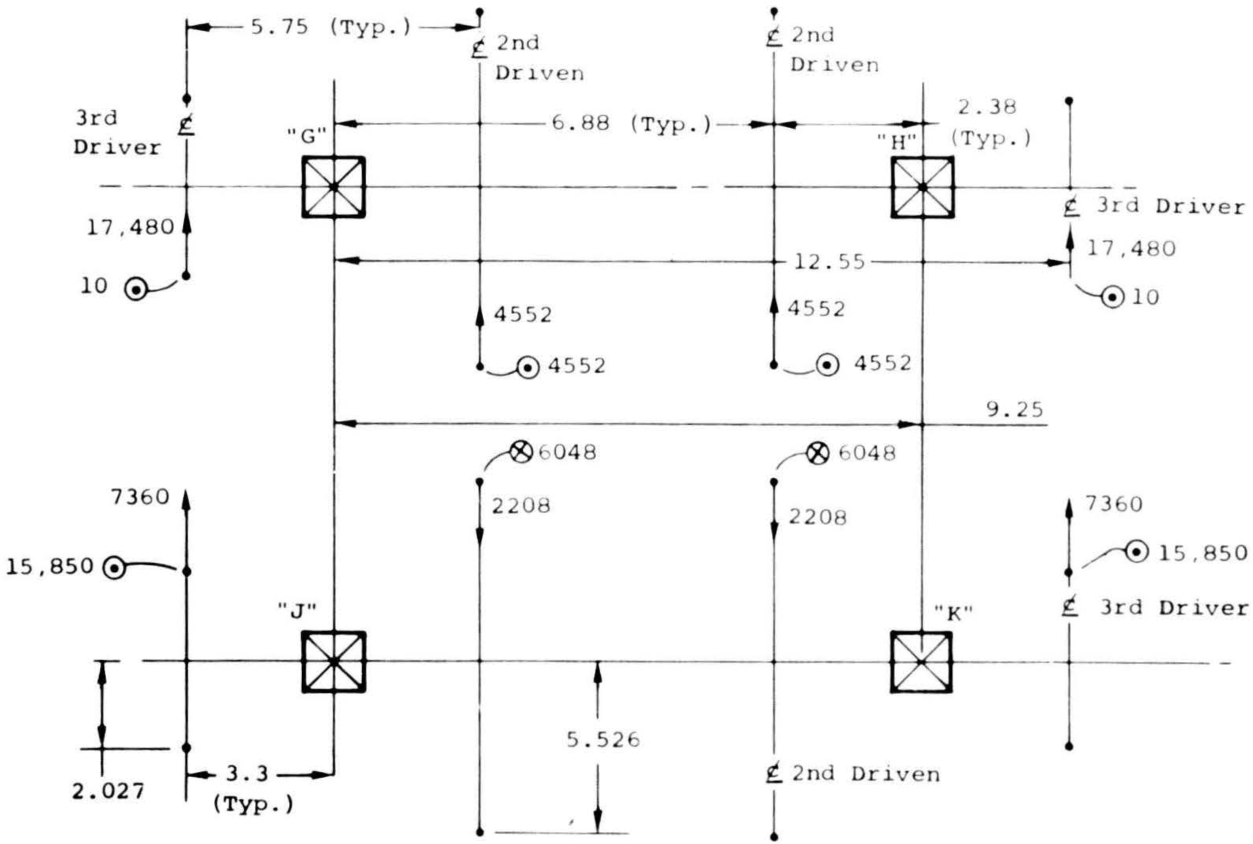
ΣH = 7,920 + 7,930 = 15,850 $\odot$

TOP MESH

ΣV = 12,420 + 5,060 = 17,480 $\uparrow$

ΣH = 7,930 - 7,920 = 10 $\odot$

BEARING LOADS - SECOND STAGE DRIVEN, THIRD STAGE DRIVER



BEARING LOADS - SECOND STAGE DRIVEN, THIRD STAGE DRIVER (CONT.)

ΣM @ G_x = 0

$$+17{,}480\ (3.3) - 4552\ (2.38) - 4552\ (6.88) - 17{,}480\ (12.55) + RH_x\ (9.25)$$

$$= +57{,}600 - 10{,}850 - 31{,}400 - 219{,}000 + 9.25\ RH_x$$

$$RH_x = \frac{203{,}650}{9.25} = 22{,}000 \text{ lb.}$$

ΣM @ G_y = 0

$$+10\ (3.3) - 4552\ (2.38) - 4552\ (6.88) - 10\ (12.55) + RH_y\ (9.25)$$

$$+33 - 10{,}850 - 31{,}400 - 125.5 + 9.25\ RH_y$$

$$RH_y = \frac{42{,}342}{9.25} = 4{,}230 \text{ lb.}$$

$$\Sigma RH = \sqrt{\overline{RH_x}^2 + \overline{RH_y}^2} = \sqrt{(22{,}000)^2 + (4230)^2}$$

$$= \sqrt{483{,}000{,}000 + 17{,}900{,}000}$$

RH = 22,570 lb.
RG = 22,570 lb.

Dynamic Capacity Required at "H" & "G"

C/P = 3.6 Shaft Speed 988 R.P.M.

3.4 x 22,570 lb. = 76,700 lb.

BEARING LOADS - SECOND STAGE DRIVEN, THIRD STAGE DRIVER (CONT.)

$\Sigma M @ J_x = 0$

$$= +7360\ (3.3) + 2208\ (2.38) + 2208\ (6.88) - 7360\ (12.55) + RK_x\ (9.25)$$

$$= +24{,}300 + 5{,}260 + 15{,}200 - 92{,}400 + 9.25\ RK_x$$

$$RK_x = \frac{47{,}640}{9.25} = 5{,}150 \text{ lb.}$$

$\Sigma M @ J_y = 0$

$$= +15{,}850\ (3.3) +6048\ (2.38) + 6048\ (6.88) - 15{,}850\ (12.55) + RK_y\ (9.25)$$

$$= +52{,}300 + 14{,}380 + 41{,}500 - 199{,}000 + 9.25\ RK_y$$

$$RK_y = \frac{90{,}820}{9.25} = 9{,}820$$

$$\Sigma RK = \sqrt{\overline{RK_x}^2 + \overline{RK_y}^2} = \sqrt{((5{,}150)^2 + (9{,}820)^2}$$

$$= \sqrt{26{,}400{,}000 + 96{,}000{,}000}$$

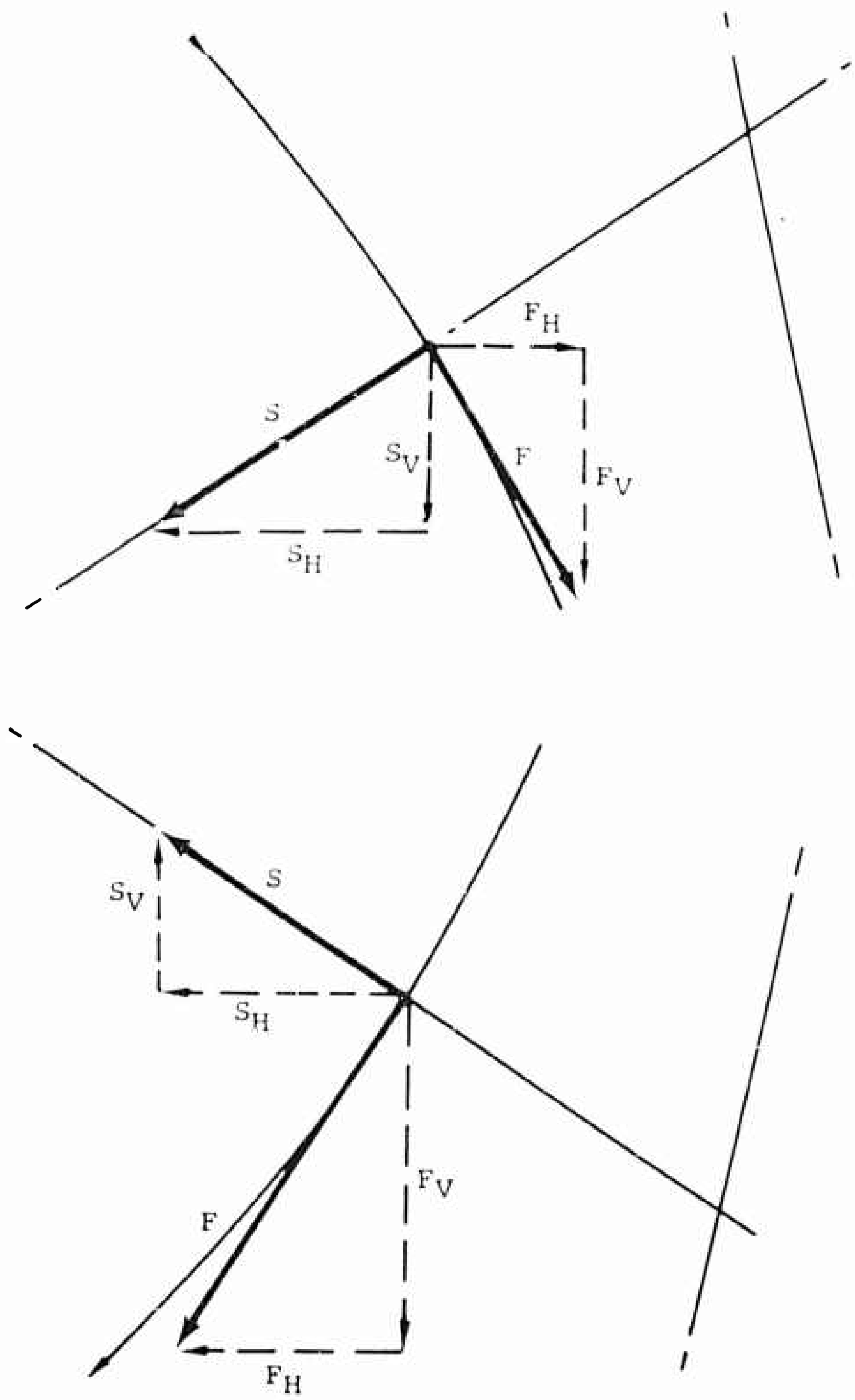
$$\begin{matrix} RK \\ RJ \end{matrix} = 11{,}090 \text{ lb.}$$

Dynamic Capacity Req'd. @ "K" & "J"

C/P = 3.6 Shaft Speed 988 R.P.M.

3.4 x 11,090 = 37,700 lb.

FORCE RESOLUTION - THIRD STAGE DRIVEN



FORCE RESOLUTION - THIRD STAGE DRIVEN (CONT.)

32° 30'

Sin = .537

Cos = .843

F_H = 14,750 x .537 = 7,920 lb.

F_V = 14,750 x .843 = 12,420 lb.

S_H = 9,420 x .843 = 7,930 lb.

S_V = 9,420 x .537 = 5,060 lb.

BOTTOM MESH

ΣV = 12,420 - 5,060 = 7360 $\downarrow$

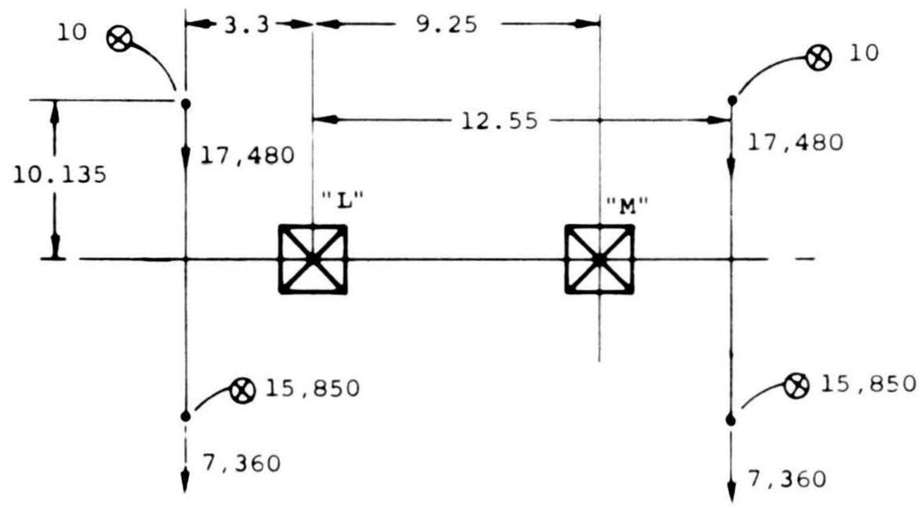
ΣH = 7920 + 7930 = 15850 $\otimes$

TOP MESH

ΣV = 12,420 + 5060 = 17,480 $\downarrow$

ΣH = 7,930 - 7920 = 10 $\otimes$

BEARING LOADS - THIRD STAGE DRIVEN



$$\Sigma M \text{ @ } L_x = 0$$

$$= -17{,}480(3.3) - 7{,}360(3.3) +$$

$$17{,}480(12.55) + 7{,}360(12.55) - RM_x(9.25)$$

$$= 57{,}600 - 24{,}300 + 219{,}000 + 92{,}400 - 9.25\ RM_x$$

$$RM_x = \frac{229{,}500}{9.25} = 24{,}800 \text{ lb.}$$

$$\Sigma M \text{ @ } L_y = 0$$

$$= -10(3.3) - 15{,}850(3.3) +$$

$$10(12.55) + 15{,}850(12.55) - RM_y(9.25)$$

$$= -33 - 52{,}300 + 125.5 + 199{,}000 - 9.25\ RM_y$$

$$RM_y = \frac{146{,}792}{9.25} = 15{,}850 \text{ lb.}$$

BEARING LOADS - THIRD STAGE DRIVEN (CONT.)

$$\Sigma RM = \sqrt{\overline{RM_x}^2 + \overline{RM_y}^2} = \sqrt{(24,800)^2 + (15,850)^2}$$

$$= \sqrt{610,000,000 + 250,000,000}$$

$$\begin{matrix} RM \\ RL \end{matrix} = 29,400 \text{ lb.}$$

Dynamic Capacity Required @ "M" & "L"

C/P = 2.1 Shaft Speed 197.5 R.P.M.

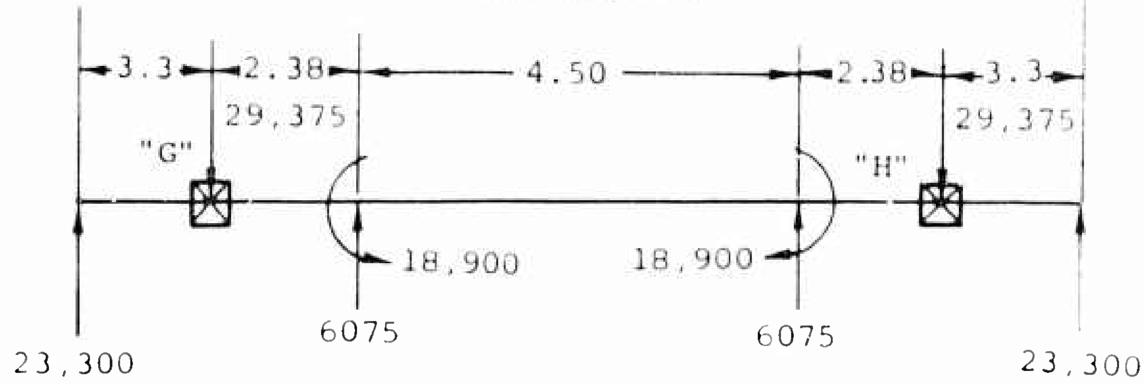
2.1 x 29,400 = 61,750 lb.

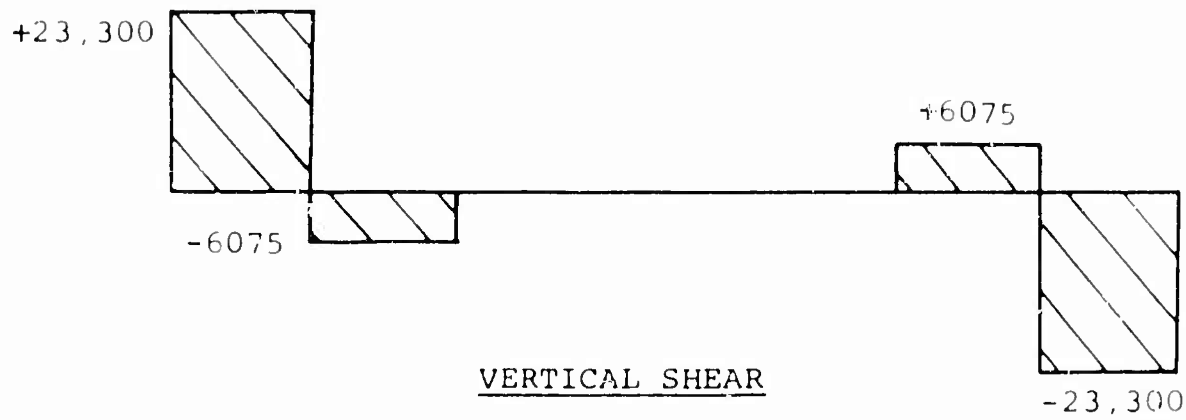
BEARING SUMMARY - 2500 HP @ 20,000 R.P.M.

Loc.	Pos.	Radial Load	Speed R.P.M.	1000-Hr. Load Ratio C/P	Dyn Cap 1000-Hr. Life	Basic Br'g. No.	Catalogue Dyn. Cap	Actual C/P	Actual Life Hrs.
Input Shaft	A	3,110	20,000	8.39	26,060	1310	22,200	7.14	620
	B	3,110	20,000	8.39	26,060	1310	22,200	7.14	620
First Driven	C	3,110	4,450	5.5	17,120	1212	17,900	5.75	1,250
	D	3,110	4,450	5.5	17,120	1212	17,900	5.75	1,250
Second Driver	E	2,784	4,450	5.5	15,320	1211	13,600	4.87	730
	F	2,784	4,450	5.5	15,320	1211	13,600	4.87	730
Third Driver	G	22,570	988	3.4	76,700	1320	72,000	3.2	840
	H	22,570	988	3.4	76,700	1320	72,000	3.2	840
	J	11,090	988	3.4	37,700	1320	72,000	6.5	8,750
	K	11,090	988	3.4	37,700	1320	72,000	6.5	8,750
Third Driven	L	29,400	197.5	2.1	61,750	1226	65,400	2.22	1,175
	M	29,400	197.5	2.1	61,750	1226	65,400	2.22	1,175

GEAR SHAFT BENDING ANALYSIS

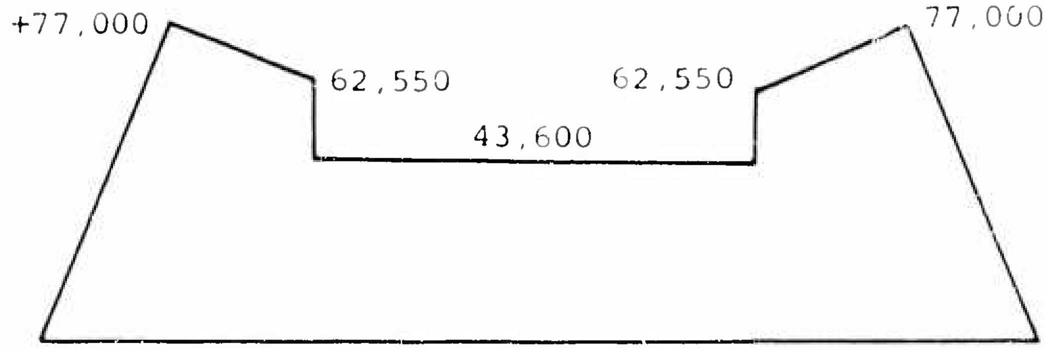
NO. 3 SHAFT





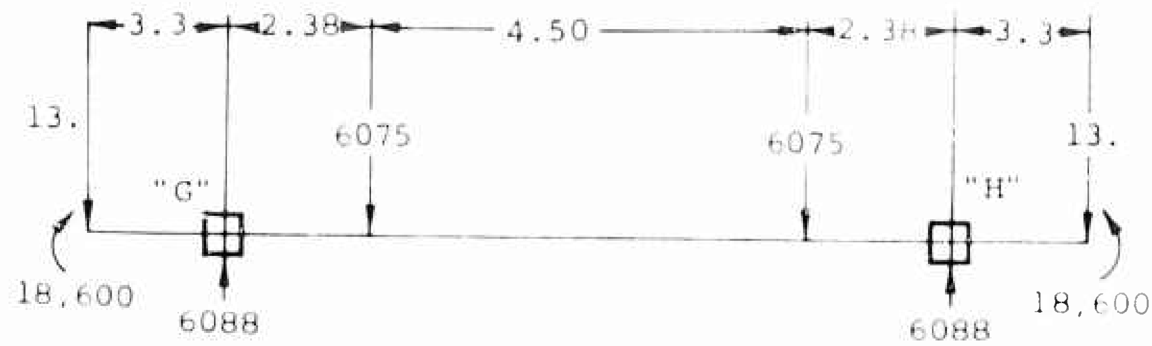
VERTICAL SHEAR

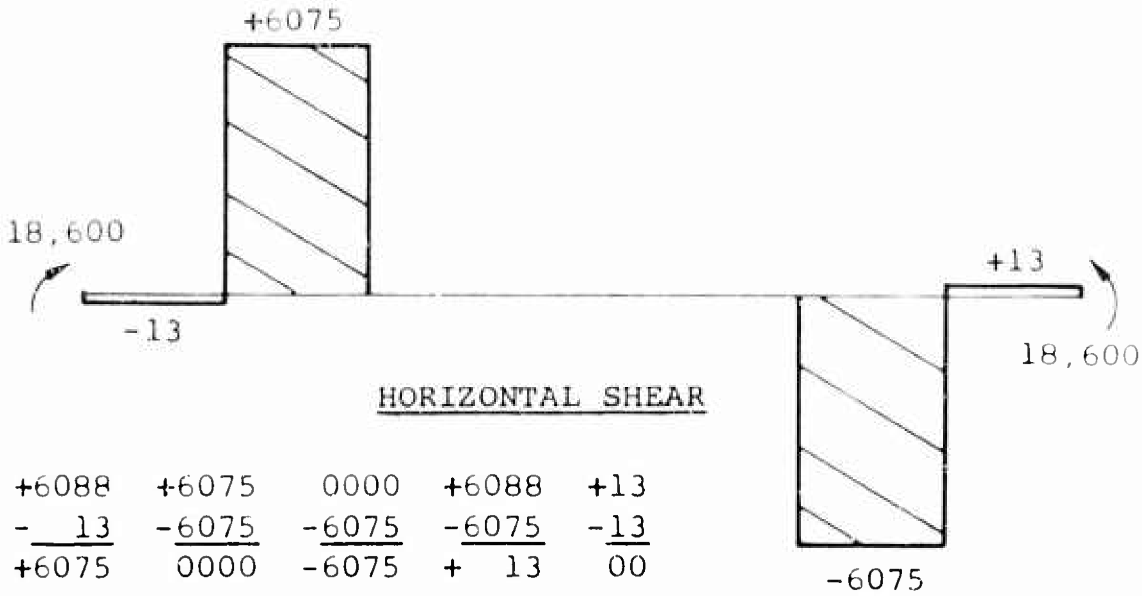
+23,300	-29,375	-6075	0000	-29,375	-23,300
	+23,300	+6075	+6075	+ 6,075	+23,300
	- 6,075	0000	+6075	-23,300	00.000



BENDING MOMENT

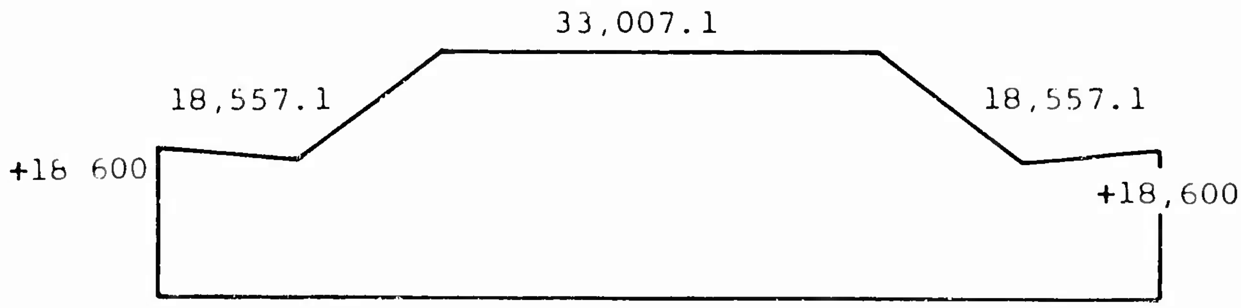
+23,300x3.3=77,000
-6075x2.38+77,000=-14,450+77,000=+62,550-18,900=43,600
+6075x2.38+62,550=+14,450+62,550=77,000
-23,300x3.3+77,000=-77,000+77,000=0





HORIZONTAL SHEAR

-13	+6088	+6075	0000	+6088	+13
	- 13	-6075	-6075	-6075	-13
	+6075	0000	-6075	+ 13	00



BENDING MOMENT

+18,600 -13 x 3.3 = +18,600 -42.9 = +18,557.1

+6075 x 2.38 + 18,557.1 = 14,450 + 18,557.1 = +33,007.1

-6075 x 2.38 + 33,007.1 = -14,450 + 33,007.1 = +18,557.1

+13 x 3.3 + 18,557.1 = +42.9 + 18,557.1 = +18,600
+ 18,600 - 18,600 = 0

$$\text{Max moment under brg.} = \sqrt{(18{,}557)^2 + (77{,}000)^2}$$

$$= \sqrt{344{,}000{,}000 + 5{,}900{,}000{,}000}$$

$$= \sqrt{6{,}244{,}000{,}000}$$

$$= 79{,}200 \text{ in.lb.}$$

$$Z = .098 \ \frac{D^4 - d^4}{D} = .098 \ \frac{(4)^4 - (1.75)^4}{4}$$

$$= .098 \ \frac{256-9.375}{4}$$

$$= .098 \ \frac{246.625}{4} = .098 \times 61.7$$

$$Z = 6.05$$

$$S = \frac{Mb}{Z} = \frac{79{,}200}{6.05} = 13{,}100 \text{ P.S.I.}$$

THIRD STAGE PINION GEAR SPLINE STRESSES

Full Depth Spline

Pitch Diameter	2.625
Spline Length	2.38
Torque	40,000 in.lb.

fbr Compressive Stress

$$fbr = \frac{T}{D^2 \times \ell}$$

$$= \frac{40{,}000 \text{ in.lb.}}{(2.625)^2 \times 2.38}$$

$$= \frac{40\,000}{16.4}$$

$$fbr = 2440 \text{ P.S.I.}$$

APPENDIX C

CONFORMAL AND PLANETARY WEIGHT STUDIES

Weight Tabulation Novikov Transmission
2,500 HP, 20,000 RPM 101.25 Ratio

1) Gear Weights

a) Third Stage Driven

$$\begin{aligned} wt &= .3(FD^2)^{.8} \\ &= .3 \times (1670)^{.8} \\ &= .3 \times 378.6 \\ wt &= 113.68 \text{ lb.} \end{aligned}$$

$$\begin{aligned} FD^2 &= 4.05\ (20.27)^2 \\ &= 4.05 \times 412 \\ FD^2 &= 1670 \end{aligned}$$

$$\text{Log } 1670 = 3.22272$$

$$\begin{array}{r} 3.22272 \\ \underline{.8} \\ 2.578176 \end{array}$$

$$= \begin{array}{r} 378.6 \\ \underline{.3} \\ 113.58 \end{array}$$

2 Gears @ 114 lb. ea.

Total Wt = 228 lb. (Third Stage Driven)

---

b) Third Stage Driver

$$\begin{aligned} wt &= .3\ (FD^2)^{.8} \\ &= .3\ (66.6)^{.8} \\ &= .3 \times 28.76 \\ wt &= 8.6 \text{ lb.} \end{aligned}$$

$$\begin{aligned} FD^2 &= 4.05\ (4.05)^2 \\ &= 4.05 \times 16.42 \\ FD^2 &= 66.6 \end{aligned}$$

$$\text{Log } 66.6 = 1.82347$$

$$\begin{array}{r} 1.82347 \\ \underline{.8} \\ 1.458776 \end{array}$$

$$= \begin{array}{r} 28.76 \\ \underline{.3} \\ 8.628 \end{array}$$

4 Gears @ 8.6 lb. ea.

Total Wt = 34.4 lb. (Third Stage Driver)

c) <u>Second Stage Driven</u>

wt	$= .3\ (FD^2)^{.8}$	FD^2	$= 2.45\ (11.06)^2$
	$= .3\ (303)^{.8}$		$= 2.45 \times 123.8$
	$= .3 \times 96.67$	FD^2	$= 303$
wt	$= 28$ lb.		
		Log 303	2.48144
			.8
			1.985152
			96.67
			.3
			28.001

4 Gears @ 28 lb. ea.

<u>Total Wt</u> = 112 lb. (Second Stage Driven)

---

d) <u>Second Stage Driver</u>

wt	$= .3\ (FD^2)^{.8}$	FD^2	$= 2.45\ (2.45)^2$
	$= .3\ (14.8)^{.8}$		$= 2.45 \times 6.05$
	$= .3 \times 8.634$	FD^2	$= 14.8$
wt	$= 2.6$ lb.		
		Log 14.8	= 1.17026
			.8
			.936208
			= 8.634
			.3
			2.5902

2 Gears @ 2.6 lb. ea.

<u>Total Wt</u> = 5.2 lb. (Second Stage Driver)

e) <u>First Stage Driven</u>

wt $.3\ (FD^2)^{.8}$ $\qquad$ FD^2 $2.2\ (9.9)^2$

$.3\ (216)^{.8}$ $\qquad$ 2.2×98

$.3 \times 73.72$ $\qquad$ FD^2 216

wt 22.12 lb.

Log 216 $\qquad$ 2.33445

2.33445
.8
1.867560

73.72
.3
22.116

1 Gear @ 22.12 lb.

<u>Total Wt</u> 22.12 lb. (First Stage Driven)

f) <u>First Stage Driver</u>

wt. $.3\ (FD^2)^{.8}$ $\qquad$ FD^2 $2.2\ (2.2)^2$

$.3\ (12.41)^{.8}$ $\qquad$ 2.2×5.64

$= .3 \times 7.499$ $\qquad$ FD^2 12.41

wt 2.25 lb. $\qquad$ Log 12.41 $\qquad$ 1.09377

1.09377
.8
.875016

7.499
.3
2.2497

1 Gear @ 2.25 lb.

<u>Total Wt</u> 2.25 lb. (First Stage Driver)

2) <u>Gear Shaft Weights</u>

a) <u>No. 1 Shaft</u>

$A = .78D^2 = .78 \times (2.5)^2 = .78 \times 6.25 = 4.87\ in^2$
$V = 4.87\ in^2 \times 7.5 = 36.6\ in^3$
$A = .78D^2 = .78\ (1.25)^2 = .78 \times 1.565 = 1.22\ in^2$
$V = 1.22 \times 7.5 = 9.15\ in^3$

$\text{Total } V = 36.6\ in^3 - 9.15\ in^3$

$= 26.45\ in^3$

$wt = 26.45\ in^3 \times .3$

$= 7.94\ lb.$ (No. 1 Shaft)

b) <u>No. 2 Shaft</u>

$A = .78D^2 = .78(2)^2 = .78 \times 4 = 3.12\ in^2$
$V = 3.12\ in^2 \times 8 = 24.96\ in^3$
$A = .78D^2 = .78(1)^2 = .78 \times 1 = .78\ in^2$
$V = .78\ in^2 \times 8 = 6.24\ in^3$

$\text{Total } V = 24.96 - 6.24$

$= 18.72\ in^3$

$wt = 18.72\ in^3 \times .3$

$wt = 5.62\ lb.$ (No. 2 Shaft)

c) <u>No. 3 Shaft</u>

A $= .78D^2 = .78(4)^2 = .78 \times 16 = 12.5\ in^2$
V $= 12.5\ in^2 \times 18 = 225\ in^3$
A $= .78D^2 = .78(2)^2 = .78 \times 4 = 3.12\ in^2$
V $= 3.12\ in^2 \times 18 = 56\ in^3$

Total V $= 225\ in^3 - 56\ in^3$

$= 169\ in^3$

wt $= 169\ in^3 \times .3$

wt = 50.7 lb. (No. 3 Shaft)

---

d) <u>No. 4 Shaft</u>

Same as No. 3 Shaft

wt = 50.7 lb. (No. 4 Shaft)

---

e) <u>No. 5 Shaft</u>

A $= .78D^2 = .78(5)^2 = .78 \times 25 = 19.5\ in^2$
V $= 19.5\ in^2 \times 12.75 = 248\ in^3$
A $= .78D^2 = .78(3.5)^2 = .78 \times 12.2 = 9.5\ in^2$
V $= 9.5\ in^2 \times 12.75 = 121\ in^3$

Total V $= 248\ in^3 - 121\ in^3$

$= 127\ in^3$

wt $= 127\ in^3 \times .3$

wt = 38.1 lb. (No. 5 Shaft)

3) Bearing Weights

a) No. 5 Shaft

2 Bearings @ 15.7 lb. ea.
2 x 15.7 = 31.4 lb.

Revise to make equal to HC-1B fwd rotor shaft bearings, 49.2 lb.

Total Wt = 49.2 lb. (No. 5 Shaft)
Brgs.

---

b) No. 4 Shaft

2 Bearings @ 18.5 lb. ea.

Total Wt = 37.0 lb. (No. 4 Shaft)
Brgs.

---

c) No. 3 Shaft

2 Bearings @ 18.5 lb. ea.

Total Wt = 37.0 lb. (No. 3 Shaft)
Brgs.

---

d) No. 2 Shaft

3 Bearings, 1.65 lb., 2.8 lb., 2.6 lb.

Total Wt = 7.05 lb. (No. 2 Shaft)
Brgs

e) No. 1 Shaft

3 Bearings - 4.5 lb., 3.6 lb., 4.0 lb.

Total Wt = 12.1 lb. (No. 1 Shaft)
Brgs.

4) Weight Summation

a) Gear Weights =

228.0	Third
34.4	
112.0	Second
5.2	
22.12	First
2.25	
403.97	

Total Gear Wt = 403.97 lb.

b) Gear Shaft Weights =

7.94	- No. 1
5.62	- No. 2
50.7	- No. 3
50.7	- No. 4
38.1	- No. 5
153.06	

Total Gear Shaft Wt = 153.06 lb.

c) Bearing Weights =

49.2	- No. 5
37.0	- No. 4
37.0	- No. 3
7.05	- No. 2
12.1	- No. 1
142.35	

Total Bearing Wt = 142.35 lb.

4) <u>Weight Summation</u> (Con'.)

Σ a + b + c

699.38 lb.

Assume 20% of above weight as a housing wt

700 x .2 140 lb.

Assume 8% for misc. (hardware, spacers, locknuts, shims, covers)

700 x .08 56 lb.

$$\begin{array}{lr} & 699.38 \\ & 140.0 \\ & \underline{56.0} \\ \text{Total Transmission Wt} & 895.38 \text{ lb.} \end{array}$$

5) <u>% of Total Transmission Weight</u>

Gears & Gear Shafts

$\frac{557}{895}$ 61.9%

Housings

$= \frac{140}{895}$ 15.6%

Bearings

$= \frac{142.35}{895}$ 15.9%

Weight Comparison - 2 Planetary Stages (17.4:1 Ratio)
HC-1B Fwd XMSN vs 2nd + 3rd Stages (22.5:1) Novikov

HC-1B Fwd

First Stage		127.3
Second Stage		214.8
1060 Rotor Shaft (40% 117 lb.)		47.0
1088 Housing		100.
S144 Brg		42.4
1041 Housing (50% 58 lb.)		29.0
	Total	560.5 lb.

Novikov

Third Stage		349.7
Second Stage		298.22
Housing (60% 140 lb.)		84.
	Total	731.92 lb.
	Δ Wt =	171.42 lb.

Weight Tabulation Novikov Transmission 1,500 HP, 25,000 RPM
101.25 Ratio

1) Gear Weights

a) Third Stage Driven

$$wt = .3\ (FD^2)^{.8} \qquad FD^2 = 3.17 \times (15.88)^2$$
$$= .3\ (798)^{.8} \qquad 3.17 \times 252$$
$$.3 \times 209.7 \qquad FD^2 = 798$$
$$wt = 62.91 \text{ lb.}$$

Log 798 = 2.90200
.8
2.321600

209.7
.3
62.91

2 Gears @ 62.91 lb. ea.

Total Wt = 125.82 lb. (Third Stage Driven)

b) Third Stage Driver

$$wt = .3\ (FD^2)^{.8} \qquad FD^2 = 3.17\ (3.17)^2$$
$$.3\ (31.84)^{.8} \qquad 3.17 \times 10.05$$
$$.3 \times 15.94 \qquad FD^2 = 31.84$$
$$wt = 4.78 \text{ lb.}$$

Log 31.84 = 1.50297
.8
1.202376

15.94
.3
4.782

4 Gears @ 4.78 lb. ea.

Total Wt = 19.12 lb. (Third Driver)

c) <u>Second Stage Driven</u>

$wt = .3\,(FD^2)^{.8}$

$= .3\,(143.2)^{.8}$

$= .3 \times 53.06$

$wt = 15.92$ lb.

$FD^2 = 1.92\,(8.64)^2$

$= 1.92 \times 74.6$

$FD^2 = 143.2$

$\text{Log } 143.2 = 2.15594$

$\times .8$

1.724752

53.06

$\times .3$

15.918

4 Gears @ 15.92 lb. ea.

<u>Total Wt</u> = 63.68 lb. (Second Driven)

---

d) <u>Second Stage Driver</u>

$wt = .3\,(FD^2)^{.8}$

$= .3\,(7.08)^{.8}$

$= 3 \times 4.787$

$wt = 1.44$ lb.

$FD^2 = 1.92\,(1.92)^2$

$= 1.92 \times 3.69$

$FD^2 = 7.08$

$\text{Log } 7.08 = 0.85003$

$\times .8$

$.680024$

$= 4.787$

$\times .3$

1.4361

2 Gears @ 1.44 lb. ea.

<u>Total Wt</u> = 2.88 lb. (Second Driver)

e) <u>First Stage Driven</u>

$wt = .3\ (FD^2)^{.8}$

$= .3\ (118.1)^{.8}$

$= .3 \times 45.48$

$wt = 13.64$ lb.

$FD^2 = 1.8\ (8.1)^2$

$= 1.8 \times 65.61$

$FD^2 = 118.1$

Log 118.1 = 2.07225

$$\begin{array}{r} 2.07225 \\ \underline{.8} \\ 1.657800 \end{array}$$

$$\begin{array}{r} 45.48 \\ \underline{.3} \\ 13.644 \end{array}$$

1 Gear @ 13.64 lb.

<u>Total Wt</u> = 13.64 lb. (First Driven)

---

f) <u>First Stage Driver</u>

$wt = .3\ (FD^2)^{.8}$

$= .3\ (5.832)^{.8}$

$= .3 \times 4.099$

$wt = 1.23$ lb.

$FD^2 = 1.8\ (1.8)^2$

$= 1.8 \times 3.24$

$FD^2 = 5.832$

Log 5.832 = 0.76582

$$\begin{array}{r} 0.76582 \\ \underline{.8} \\ .612656 \end{array}$$

$$\begin{array}{r} 4.099 \\ \underline{.3} \\ 1.2297 \end{array}$$

1 Gear @ 1.23 lb.

<u>Total Wt</u> = 1.23 lb. (First Driver)

2) <u>Gear Shaft Weights</u>

a) <u>No. 1 Shaft</u>

$A = .78D^2 = .78(1.5)^2 = .78 \times 2.25 = 1.755\ in^2$
$V = 1.755\ in^2 \times 7.1 = 12.46\ in^3$
$A = .78D^2 = .78(.88)^2 = .78 \times .774 = .603\ in^2$
$V = .603 \times 7.1 = 4.28\ in^3$

Total $V = 12.46\ in^3 - 4.28\ in^3$

$= 8.18\ in^3$

$wt = 8.18\ in^3 \times .3$

$wt = 2.45$ lb. (No. 1 Shaft)

---

b) <u>No. 2 Shaft</u>

$A = .78D^2 = .78(1.5)^2 = .78 \times 2.25 = 1.755\ in^2$
$V = 1.755\ in^2 \times 5.5 = 9.66\ in^3$
$A = .78D^2 = .78(.62)^2 = .78 \times .384 = .30\ in^2$
$V = .30\ in^2 \times 5.5 = 1.65\ in^3$

Total $V = 9.66\ in^3 - 1.65$

$= 8.01\ in^3$

$wt = 8.01 \times .3$

$wt = 2.40$ lb. (No. 2 Shaft)

c) No. 3 Shaft

A = $.78D^2$ = $.78(2.25)^2$ = .78 x 5.06 = 3.95 in^2
V = 3.95 in^2 x 6.62 = 26.2 in^3
A = $.78D^2$ = $.78(1.0)^2$ = .78 x 1.0 = .78 in^2
V = .78 x 6.62 = 5.16 in^3

Total V = 26.2 - 5.16

= 21.04 in^3

wt = 21.04 in^3 x .3

wt = 6.31 lb. (No. 3 Shaft

---

d) No. 4 Shaft

Same as No. 3

wt = 6.3 lb. (No. 4 Shaft)

---

e) No. 5 Shaft

A = $.78D^2$ = $.78(5.88)^2$ = .78 x 34.6 = 27 in^2
V = 27 x 10.5 = 284 in^3
A = $.78D^2$ = $.78(4.75)^2$ = .78 x 22.6 = 17.6
V = 17.6 x 10.5 = 180.5

Total V = 284 - 180.5

= 103.5 in^3

wt = 103.5 x .3

wt = 31.05 lb. (No. 5 Shaft)

3) Bearing Weights

a) No. 5 Shaft

2 Rollway 1930 bearings @ 7.00 lb. ea.

Assume addition of 20% to bearing weights to compensate for rotor loads.

14.0 lb. x .2 = 2.8 lb.

Total Bearing Weight = 16.8 lb. (No. 5 Shaft)

b) No. 4 Shaft

2 Rollway 1313 bearings @ 5.5 lb. ea.

Total Bearing Weight = 11.0 lb. (No. 4 Shaft)

c) No. 3 Shaft

2 Rollway 1313 bearings @ 5.5 lb. ea.

Total Bearing Weight = 11.0 lb. (No. 3 Shaft)

d) No. 2 Shaft

1 Rollway 1306 @ .88 lb., 1 Rollway 1308 @ 1.65 lb.

1 MRC 9308-U @ 1.5 lb.

Total Bearing Weight = 4.03 lb. (No. 2 Shaft)

e) No. 1 Shaft

2 Rollway 1308 bearings @ 1.65 lb. ea.

1 MRC 9308-U bearings @ 1.5 lb. ea.

Total Bearing Weight = 4.8 lb. (No. 1 Shaft)

4) Weight Summation

Σ 1 + 2 + 3 = 226.37 lb. + 48.52 lb. + 47.63 lb.

= 322.52 lb.

Assume 20% of above weight for housing wt.

323 lb x .2 = 65 lb.

Assume 8% for misc. (hardware, shims, locknuts. covers, etc.).

323 x .08 = 26 lb.

Total Transmission Weight = 322.52 + 65 + 26

= 413.52 lb. (Total Transmission Weight)

---

5) Percent Weight Distribution

Gears & Gear Shafts = $\frac{275}{414}$ = 66.5%

Housing = $\frac{65}{414}$ = 15.7%

Bearings = $\frac{48.52}{414}$ = 11.9%

6) <u>Weight - Second & Third Stages Only</u>

Third Stage	192.79
Second Stage	107.61
60% for housing - 65 lb.	39.00
Total Wt (2nd + 3rd Stages)	339.40 lb.

Weight Tabulation Novikov Transmission 250 HP, 35,000 RPM
101.25 Ratio

1) Gear Weights

a) Third Stage Driven

$wt = .3(FD^2)^{.8}$	$FD^2 = 1.56(7.82)^2$
$= .3(95.5)^{.8}$	$= 1.56 \times 61.2$
$= .3 \times 38.37$	$FD^2 = 95.5$
$wt = 11.51$ lb.	$\text{Log } 95.5 = 1.98000$
	$\times .8$
	1.584000
	$= 38.37$
	$\times .3$
	11.511

2 Gears @ 11.51 lb. ea.

Total Wt = 23.02 lb. (Third Driven)

---

b) Third Stage Driver

$wt = .3(FD^2)^{.8}$	$FD^2 = 1.56(1.56)^2$
$= .3(3.81)^{.8}$	$= 1.56 \times 2.44$
$= .3(2.916)$	$FD^2 = 3.81$
$Wt = .87$ lb.	$\text{Log } 3.81 = 0.58092$
	$\times .8$
	$.464736$
	$= 2.916$
	$\times .3$
	$.8748$

4 Gears @ .87 lb. ea.

Total Wt = 3.5 lb. (Third Driver)

c) <u>Second Stage Driven</u>

$$wt = .3(FD^2)^{.8}$$
$$= .3(17.26)^{.8}$$
$$= .3 \times 9.764$$
$$wt = 2.93 \text{ lb.}$$

$$FD^2 = .95(4.26)^2$$
$$= .95 \times 18.16$$
$$FD^2 = 17.26$$

$$\text{Log } 17.26 = \begin{array}{r} 1.23704 \\ \underline{.8} \\ .989632 \end{array}$$

$$= \begin{array}{r} 9.764 \\ \underline{.3} \\ 2.9292 \end{array}$$

4 Gears @ 2.93 lb. ea.

Total Wt = 11.72 lb. (Second Driven)

---

d) <u>Second Stage Driver</u>

$$wt = .3(FD^2)^{.8}$$
$$= .3(.848)^{.8}$$
$$= .3 \times .8764$$
$$wt = .26 \text{ lb.}$$

$$FD^2 = .95(.95)^2$$
$$= .95 \times .892$$
$$FD^2 = .848$$

$$\text{Log } .848 = \begin{array}{rr} 9.92840 & -10 \\ \underline{.8} & \\ 7.942720 & -8 \end{array}$$

$$= \begin{array}{r} .8764 \\ \underline{.3} \\ .26292 \end{array}$$

2 Gears @ .26 lb. ea.

Total Wt = .52 lb. (Second Driver)

e) <u>First Stage Driven</u>

$wt = .3(FD^2)^{.8}$ | $FD^2 = .90(4.05)^2$

$= .3(14.65)^{.8}$ | $= .90 \times 16.8$

$= .3 \times 8.564$ | $FD^2 = 14.65$

$wt = 2.57$ lb. | $\text{Log } 14.65 = 1.16584$

$$\begin{array}{r} 1.16584 \\ .8 \\ \hline .932672 \end{array}$$

$$= \begin{array}{r} 8.564 \\ .3 \\ \hline 2.5692 \end{array}$$

1 Gear @ 2.57 lb.

Total Wt = 2.57 lb. ,(First Driven)

---

f) <u>First Stage Driver</u>

$wt = .3(FD^2)^{.8}$ | $FD^2 = .9(.90)^2$

$= .3(.729)^{.8}$ | $= .9 \times .81$

$= .3 \times .7766$ | $FD^2 = .729$

$wt = .23$ lb. | $\text{Log } .729 = 9.86273 \quad -10$

$$\begin{array}{rr} 9.86273 & -10 \\ .8 & \\ \hline 7.890184 & -8 \end{array}$$

$$= \begin{array}{r} .7766 \\ .3 \\ \hline .23298 \end{array}$$

1 Gear @ .23 lb.

Total Wt = .23 lb. (First Driver)

2) Gear Shaft Weights

a) No. 1 Shaft

$A = .78D^2 = .78(.94)^2 = .78 \times .885 = .687$ sq. in.

$V = .687$ sq. in. x 4.0 = 2.748 cu. in.

$A = .78D^2 = .78(.38)^2 = .78 \times .140 = .109$ sq. in.

$V = .109$ sq. in. x 4.0 = 436 cu. in.

Total V = 2.748 cu. in. - .436 cu. in.

= 2.312 cu. in.

wt = 2.312 cu. in. x .3

wt = .69 lb. (No. 1 Shaft)

---

b) No. 2 Shaft

$A = .78D^2 = .78(1.0)^2 = .78 \times 1 = .78$ sq. in.

$V = .78$ sq. in. x 8.0 = 6.24 cu. in.

$A = .78D^2 = .78(.38)^2 = .78 \times .140 = .109$ sq. in.

$V = .109$ sq. in. x 3.88 = .423 cu. in.

Total V = 6.24 cu. in. - .423 cu. in.

= 5.82 cu. in.

wt = 5.82 cu. in. x .3

wt = 1.75 lb. (No. 2 Shaft)

c) <u>No. 3 Shaft</u>

$A = 78D^2 = .78(2.0)^2 = .78 \times 4 = 3.12$ sq. in.

$V = 3.12$ sq. in. $\times 8.0 = 24.96$ cu. in.

$A = .78D^2 = .78(.75)^2 = .78 \times .56 = .437$ sq. in.

$V = .437$ sq. in. $\times 8.0 = 3.496$ cu. in.

Total $V = 24.96$ cu. in. $- 3.496$ cu. in.

$= 21.464$ cu. in.

$Wt = 21.464$ cu. in. $\times .3$

$Wt = 6.44$ lb. (No. 3 Shaft)

---

d) <u>No. 4 Shaft</u>

Same as No. 3

$Wt = 6.44$ lb. (No. 4 Shaft)

---

e) <u>No. 5 Shaft</u> (Between Gears)

$A = .78D^2 = .78 \times (3.12)^2 = .78 \times 9.74 = 7.59$ sq. in.

$V = 7.59$ sq. in. $\times 9.0 = 68.31$ cu. in.

$A = .78D^2 = .78(2.5)^2 = .78 \times 6.25 = 4.87$ sq. in.

$V = 4.87$ sq. in. $\times 7.0 = 34.09$ cu. in.

Total $V = 68.31$ cu. in. $- 34.09$ cu. in.

$= 34.22$ cu. in.

$Wt = 34.22$ cu. in. $\times .3$

$Wt = 10.27$ lb. (No. 5 Shaft)

3) Bearing Weights

a) No. 5 Shaft

2 Rollway 1016 Bearings @ 2.25 lb. ea.

Assume addition of 20% to bearing weight to compensate for rotor loads (5.0 lb. x .2 = 1.0 lb.)

Total Bearing Weight = 6.0 lb. (No. 5 Shaft)

---

b) No. 4 Shaft

2 Rollway 1308 Bearings (Outer Race + Rollers) @ 1.65 lb. ea.

Total Bearing Weight = 3.30 lb. (No. 4 Shaft)

---

c) No. 3 Shaft

Same as No. 4

Total Bearing Weight = 3.30 lb. (No. 3 Shaft)

---

d) No. 2 Shaft

2 Rollway 1003 Bearings @ .10 lb.

2 Rollway 1005 Bearings @ .21 lb.

1 MRC 9304-U Bearing @ .33 lb.

Total Bearing Weight = .95 lb. (No. 2 Shaft)

e) <u>No. 1 Shaft</u>

2 Rollway 1204 Bearings @ .28 lb. ea.

1 MRC 9304-U Bearing @ .33 lb.

Total Bearing Weight = .99 lb. (No. 1 Shaft)

---

4) <u>Weight Summation</u>

Σ 1 + 2 + 3 = 41.56 + 25.59 + 14.74

Σ = 81.89 lb.

Assume 20% for Housing Weight = .2 x 82 lb. = 16.4 lb.

Assume 8% for Misc. = .08 x 82 lb. = 7 lb.

Total Transmission Wt = 81.89 lb. + 16.4 lb. + 7 lb.

= 105.29 lb. (Total Transmission Weight)

---

5) <u>% Weight Distribution</u>

$$\text{Gears \& Gear Shafts} = \frac{67.15}{105.29} = 63.5\%$$

$$\text{Housing} = \frac{16.4}{105.29} = 15.5\%$$

$$\text{Bearings} = \frac{14.74}{105.29} = 14\%$$

---

6) <u>Weight of Second & Third Stages Only</u>

Third Stage	42.79
Second Stage	34.21
60%, 16.4 lb. Housing	10.00
Total	87.00

WEIGHT COMPUTATIONS WITH FORMULAS & CURVES FROM
R. WILLIS - PRODUCT DESIGN 1-21-63

3-Stage Planetary (First Stage Star), Output Torque 800,000
In.Lb. 2500 HP, 25 to 1 Ratio

FIRST STAGE

Ratio 2 to 1

$$C = \frac{2T}{K} = \frac{2 \times 32,000}{1000} = 64$$

$$\text{Torque} = \frac{800,000}{25} = 32,000 \text{ in.lb.}$$

$$\Sigma FD^2/C = 3.0$$

$$\Sigma FD^2 = 3.0 \times 64 = 192$$

$$Wt = 192 \times .3 = 57.6 \text{ lb.}$$

SECOND STAGE

Ratio 4 to 1

$$C = \frac{2T}{K} = \frac{2 \times 64,000}{1000} = 128$$

$$\text{Torque} = 32,000 \times 2 = 64,000 \text{ in.lb.}$$

$$\Sigma FD^2/C = 4.0$$

$$\Sigma FD^2 = 4.0 \times 128 = 512$$

$$Wt = 512 \times .3 = 153.6 \text{ lb.}$$

THIRD STAGE

Ratio 3.125 to 1

$$C = \frac{2T}{K} = \frac{2 \times 256,000}{1000} = 512$$

$$\text{Torque} = 64,000 \times 4 = 256,000 \text{ in.lb.}$$

$$\Sigma FD^2/C = 2.0$$

$$\Sigma FD^2 = 2.0 \times 512 = 1024$$

$$Wt = 1024 \times .3 = 307.2 \text{ lb.}$$

Total Wt Transmission = 307.2 lb. + 153.6 lb. + 57.6 lb.

= 518.4 lb. (Total Weight)

3-Stage Planetary (First Stage Star), Output Torque 800,000 In.Lb. 2500 HP, 50 to 1 Ratio

FIRST STAGE

Ratio = 3 to 1

$$C = \frac{2T}{K} = \frac{2 \times 16{,}000}{1000} = 32$$

$$\text{Torque} = \frac{800{,}000}{50} = 16{,}000 \text{ in.lb.}$$

$$\Sigma FD^2/C = 3.75$$

$$\Sigma FD^2 = 3.75 \times 32 = 120$$

$$Wt = 120 \times .3 = 36.0 \text{ lb.}$$

SECOND STAGE

Ratio = 4.8 to 1

$$C = \frac{2T}{K} = \frac{2 \times 48{,}000}{1000} = 96$$

$$\text{Torque} = 16{,}000 \times 3 = 48{,}000 \text{ in.lb.}$$

$$\Sigma FD^2/C = 6.0$$

$$\Sigma FD^2 = 6.0 \times 96 = 576$$

$$Wt = 576 \times .3 = 172.8 \text{ lb.}$$

THIRD STAGE — Ratio = 3.475 to 1

$$C = \frac{2T}{K} = \frac{2 \times 230{,}000}{1000} = 460$$

Torque = 48,000 x 4.8

= 230,000 in.lb.

$$\Sigma FD^2/C = 2.5$$

$$\Sigma FD^2 = 2.5 \times 460 = 1150$$

$$Wt = 1150 \times .3 = 365.0 \text{ lb.}$$

Total Wt Transmission = 365 lb. + 172.8 lb. + 36 lb.

= 573.8 lb. (Total Weight)

3-Stage Planetary (First Stage Star), Output Torque 800,000 In.Lb. 2500 HP, 100 to 1 Ratio

FIRST STAGE — Ratio = 4.5 to 1

$$C = \frac{2T}{K} = \frac{2 \times 8{,}000}{1000} = 16$$

$$\text{Torque} = \frac{800{,}000}{100}$$

= 8,000 in.lb.

$$\Sigma FD^2/C = 9.0$$

$$\Sigma FD^2 = 9.0 \times 16 = 144$$

$$Wt = 144 \times .3 = 43.2 \text{ lb.}$$

SECOND STAGE — Ratio = 5 to 1

$$C = \frac{2T}{K} = \frac{2 \times 36{,}000}{1000} = 72$$

Torque = 8,000 x 4.5

= 36,000 in.lb.

$$\Sigma FD^2/C = 7.0$$

$$\Sigma FD^2 = 7.0 \times 72 = 504$$

$$Wt = 504 \times .3 = 151.2 \text{ lb.}$$

THIRD STAGE — Ratio 4.45 to 1

$$C = \frac{2T}{K} \quad \frac{2 \times 180{,}000}{1000} = 360 \qquad \text{Torque} = 36{,}000 \times 5$$

$$\Sigma FD^2/C = 5.3 \qquad = 180{,}000 \text{ in.lb.}$$

$$\Sigma FD^2 = 5.3 \times 360 = 1910$$

$$Wt = 1910 \times .3 = 573.0 \text{ lb.}$$

Total Wt Transmission = 573.0 lb. + 151.2 + 43.2

= 767.4 lb. (Total Weight)

2-Stage Planetary, Output Torque 800,000 in.lb. 2500 HP, 12.5 to 1 Ratio

FIRST STAGE — Ratio 4 to 1

$$C = \frac{2T}{K} \quad \frac{2 \times 64{,}000}{1000} = 128 \qquad \text{Torque} = \frac{800{,}000}{12.5}$$

$$\Sigma FD^2/C = 4.0 \qquad = 64{,}000 \text{ in.lb.}$$

$$\Sigma FD^2 = 4 \times 128 = 512$$

$$Wt = 512 \times .3 = 153.6 \text{ lb.}$$

SECOND STAGE — Ratio 3.125 to 1

$$C = \frac{2T}{K} = \frac{2 \times 256{,}000}{1000} = 512$$

Torque = 64,000 x 4 = 256,000 in.lb.

$$\Sigma FD^2/C = 2.0$$

$$\Sigma FD^2 = 2.0 \times 512 = 1024$$

$$Wt = 1024 \times .3 = 307.2 \text{ lb.}$$

Total Wt Transmission = 307.2 lb. + 153.6 lb.

= 460.8 lb. (Total Weight)

2-Stage Planetary, Output Torque 800,000 In.Lb. 2500 HP, 22.5 to 1 Ratio

FIRST STAGE — Ratio 5.5 to 1

$$C = \frac{2T}{K} = \frac{2 \times 35{,}600}{1000} = 71.2$$

$$\text{Torque} = \frac{800{,}000}{22.5} = 35{,}600 \text{ in.lb.}$$

$$\Sigma FD^2/C = 9.0$$

$$\Sigma FD^2 = 9.0 \times 71.2 = 640.8$$

$$Wt = 640.8 \times .3 = 192.24 \text{ lb.}$$

SECOND STAGE — Ratio 4.1 to 1

$$C = \frac{2T}{K} = \frac{2 \times 196{,}000}{1000} = 392$$

Torque = 35,600 in.lb.

x 5.5

= 196,000 in.lb.

$$\Sigma FD^2/C = 4.0$$

$$\Sigma FD^2 = 4.0 \times 392 = 1568$$

$$Wt = 1568 \times .3 = 470.4 \text{ lb.}$$

Total Wt Transmission = 470.4 lb. + 192.24 lb.

= 662.64 lb. (Total Weight)

2-Stage Planetary, Output Torque 800,000 in.lb. 2500 HP, 17.5 to 1 Ratio

FIRST STAGE — Ratio 5 to 1

$$C = \frac{2T}{K} = \frac{2 \times 45{,}750}{1000} = 91.5 \qquad \text{Torque} = \frac{800{,}000}{17.5}$$

$$\Sigma FD^2/C = 7.0 \qquad = 45{,}750 \text{ in.lb.}$$

$$\Sigma FD^2 = 7.0 \times 91.5 = 640.5$$

$$Wt = 640.5 \times .3 = 192.15 \text{ lb.}$$

SECOND STAGE — Ratio 3.5 to 1

$$C = \frac{2T}{K} = \frac{2 \times 228{,}000}{1000} = 456 \qquad \text{Torque} = 45{,}750 \times 5$$

$$\Sigma FD^2/C = 2.75 \qquad = 228{,}000 \text{ in.lb.}$$

$$\Sigma FD^2 = 2.75 \times 456 = 1255$$

$$Wt = 1255 \times .3 = 376.5 \text{ lb.}$$

Total Wt Transmission = 376.5 lb. + 192.15 lb.

= 568.65 lb. (Total Weight)

3-Stage Planetary (First Stage Star), Output Torque 383,220 In.Lb. 1500 HP, 25 to 1 Ratio

FIRST STAGE Ratio 2 to 1

$$C = \frac{2T}{K} = \frac{2 \times 15{,}320}{1000} = 30.7 \qquad \text{Torque} = \frac{383{,}220}{25}$$

$$\Sigma\, FD^2/C = 3.0 \qquad = 15{,}320 \text{ in.lb.}$$

$$\Sigma\, FD^2 = 3.0 \times 30.7 = 92.1$$

$$Wt = 92.1 \times .3 = 27.63 \text{ lb.}$$

SECOND STAGE Ratio 4 to 1

$$C = \frac{2T}{K} = \frac{2 \times 30{,}640}{1000} = 61.28 \qquad \text{Torque} = 15{,}320 \times 2$$

$$\Sigma\, FD^2/C = 4.0 \qquad = 30{,}640 \text{ in.lb.}$$

$$\Sigma\, FD^2 = 4.0 \times 61.28 = 245.12$$

$$Wt = 245.12 \times .3 = 73.54 \text{ lb.}$$

THIRD STAGE Ratio 3.125 to 1

$$C = \frac{2T}{K} = \frac{2 \times 122{,}560}{1000} = 245.12 \qquad \text{Torque} = 30{,}640 \times 4$$

$$\Sigma\, FD^2/C = 2.0 \qquad = 122{,}560$$

$$\Sigma\, FD^2 = 2.0 \times 245.12$$

$$Wt = 245.12 \times .3 = 147.07 \text{ lb.}$$

Total Wt Transmission = 147.07 lb. + 73.54 lb. + 27.63 lb.

= 248.24 lb. (Total Weight)

COMPLETE WEIGHTS BY RATIO OF OUTPUT TORQUES

3-Stage Planetary-Output Torque 383,220 in.lb. 1500 HP

RATIO	WT.
50 to 1	274.6 lb.
100 to 1	368 lb.

2-Stage Planetary - 1500 HP, Output Torque 383,220 in.lb.

RATIO	WT.
12.5 to 1	221 lb.
17.5 to 1	272 lb.
22.5 to 1	317 lb.

3-Stage Planetary - 250 HP, Output Torque 45,750 in.lb.

RATIO	WT. *
25 to 1	54.0 lb.
50 to 1	59.8 lb.
100 to 1	80 lb.

2-Stage Planetary - 250 HP, Output Torque 45,750 in.lb.

RATIO	WT. *
12.5	48.2 lb.
17.5	59.0 lb.
22.5	69.2 lb.

* Weights ratio up by 1.83 by comparison to L.O.H. Proposal Wt of 59 lb. for transmission, 250 HP, 16 to 1 ratio.

SUMMATION OF TRANSMISSION WEIGHTS

		Dead Wt. (lb.)	HP Loss	Eff. Wt. (lb.)
2500 HP				
Novikov 3-Stage	101.25:1	896	56.3	1318
2-Stage	22.5:1	731.92	37.5	912.92
Planetary 3-Stage	25:1	518.4	84.4	1151.4
	50:1	573.8	84.4	1206.8
	100:1	767.4	84.4	1400.4
Planetary 2-Stage	12.5:1	460.8	56.3	882.8
	17.5:1	568.65	56.3	990.65
	22.5:1	662.64	56.3	1084.64
1500 HP				
Novikov 3-Stage	101.25:1	413.52	33.8	667.52
2-Stage	22.5:1	339.4	22.5	508
Planetary 3-Stage	25:1	248.24	50.7	628.24
	50:1	274.6	50.7	654.6
	100:1	368	50.7	748
Planetary 2-Stage	12.5:1	221	33.8	475
	17.5:1	272	33.8	526
	22.5:1	317	33.8	571
250 HP				
Novikov 3-Stage	101.25:1	104.55	5.63	146.75
2-Stage	22.5:1	87	3.75	115.2
Planetary 3-Stage	25:1	54.0	8.45	117.3
	50:1	59.8	8.45	123.1
	100:1	80	8.45	143.3
Planetary 2-Stage	12.5:1	48.2	5.63	90.4
	17.5:1	59.0	5.63	101.2
	22.5:1	69.2	5.63	111.4

EFFECTIVE WEIGHT CALCULATION

Power Loss = .75% Per Mesh

Planetary 1-Stage = 1.5 Meshes

Loss of 1 Horsepower = 7.5 lb.

Planetary 3-Stage = 3 x 1.5 = 4.5 x .75% = 3.375%

Planetary 2-Stage = 2 x 1.5 = 3.0 x .75% = 2.25%

Novikov 3-Stage = 3 x 1 = 3 x .75% = 2.25%

Novikov 2-Stage = 2 x 1 = 2 x .75% = 1.5%

Effective Wt = Dead Wt + (% Power Loss x HP x LB/HP)

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Vertol Division, The Boeing Co., Morton, Pa. INVESTIGATION OF THE CONFORMAL GEAR FOR HELICOPTER POWER TRANSMISSION by J. Mack, May 1964. 124 pages. Vertol Report R-345. USATRECOM TASK 1D121401A14414 (Contract DA44-177-AMC-101(T).

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